SEPTEMBER 1955

Journal
of
The Franklin Institute

1826

1955

How Commercial Titanium and Zirconium Were Born
W. J. Kroll

Matrices in Analogue Mathematical Machines
Pierre M. Honnell and Robert E. Horn

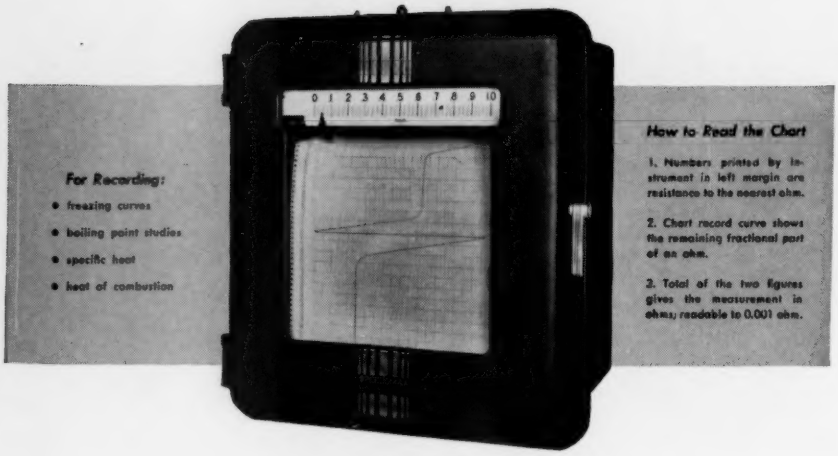
Complex Zeros of the Error Function
Herbert E. Salzer

On Electromagnetic Induction
Parry Moon and Domina Eberle Spencer

Franklin Institute:
Research Notes
Department Reports

Book Reviews
Current Topics
NBS Notes

VOL. 260 NO. 3


For Recording:
• freezing curves
• boiling point studies
• specific heat
• heat of combustion
How to Read the Chart
1. Numbers printed by instrument in left margin are resistance to the nearest ohm.
2. Chart record curve shows the remaining fractional part of an ohm.
3. Total of the two figures gives the measurement in ohms, readable to 0.001 ohm.

CAN YOU USE THIS

High-Precision Temperature Recorder

IN YOUR LABORATORY?

• Here, for the first time, is a recording instrument which not only measures temperature as precisely as your manually-operated Mueller temperature bridge, but automatically records it on a moving chart. It covers the entire platinum resistance thermometer range from −260 to +500 C . . . measures resistance change equivalent to 0.01 C. Based on a concept of Dr. Daniel R. Stull of Dow Chemical Co., this instrument helps a researcher to get the most from each working hour . . . is valuable in a variety of experiments involving precise temperature studies.

SPECIFICATIONS FOR 69812 SPEEDOMAX RECORDER

RANGE: 0 to 100 ohms, absolute, divided between slidewire and decades as in manual bridge instruments. Slidewire span is 0 to 1 ohm, readable to 0.001 ohm; extension decades are 9 (1+10) ohms.

ACCURACY OF ADJUSTMENT: ±0.01% of reading, or 0.003 ohm, whichever is greater.

CHART SPEED: 6″ per hour, nominal.

BALANCING TIME: 3 seconds nominal. Decade switching time is 3 seconds nominal per decade step.

CONSTANT TEMPERATURE CHAMBER: a thermally-insulated, thermostatically controlled housing, containing principal measuring circuit resistors.

HUMIDITY COMPENSATION: recorder-pen travel is continuously corrected for changes of chart width due to ambient humidity fluctuations.

For further facts, address our nearest office or 4992 Stenton Ave., Phila. 44, Penna.

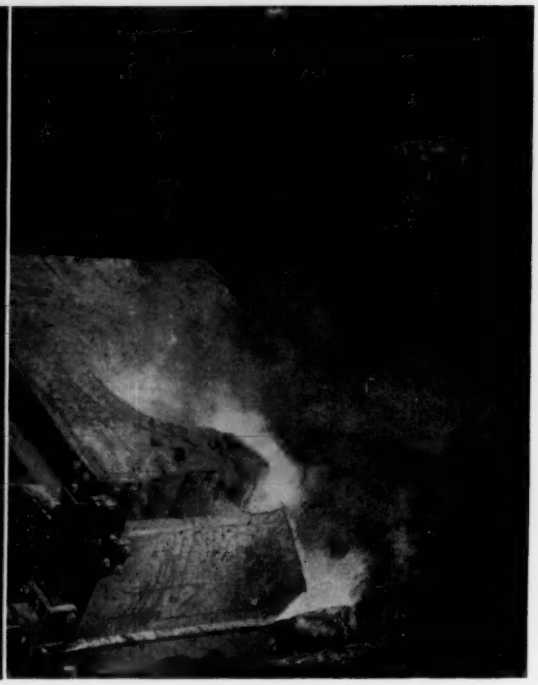
LEEDS NORTHRUP
instruments automatic controls • furnaces

INDEX TO ADVERTISERS

	PAGE
Ajax Electrothermic Corp.	ii
Almo Radio Co.	xii
Atlas Photo Engraving Co.	x
Bell Telephone Laboratories	vi
Biddle, James G. Co.	ix
Bolger-Parker Co.	x
Buten, M., & Sons	xi
Coleman, W. B., & Co.	iv
Damon & Foster	iv
Davidson, P. L.	iv
Eggly-Furlow Engineers	iv
Engineering & Refrigeration, Inc.	xi
Fehr & Johnson, Inc.	x
Fidelity-Philadelphia Trust Co.	xiv
FRANKLIN INSTITUTE, Awards	xiii
FRANKLIN INSTITUTE, Laboratories	Fourth cover page
FRANKLIN INSTITUTE, Membership	Third cover page
Greiner Co., The Emil	iii
Harris-Dechant Associates	iv
Herbach & Rademan	x
Hess & Barker	xii
Kearney Lumber Co.	x
Lancaster Press, Inc.	viii
Leeds & Northrup Co.	Second cover page
Leopold, Charles S.	iv
Levy, Max & Co.	xi
Shanahan & Co.	x
Steinberg, Albert, and Company	ix
Thomas, Arthur H., Co.	xii
Yarnall-Waring Co.	viii

Entered as second-class matter March 14, 1928, at the post office at Lancaster, Pa., under act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in paragraph (d-2), section 34.40, P. L. & R. of 1948, authorized on July 3, 1919.

600 pounds OF BRONZE melted in 23 minutes

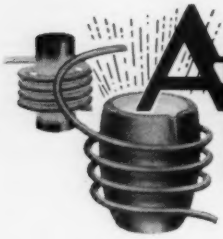


This steel and non-ferrous centrifugal-casting foundry saves more than time through the use of Ajax-Northrup induction furnaces. Freedom from contamination and almost complete metal recovery are direct results of the high speed characteristic of the Ajax-Northrup melting principle. Electrical energy, used as the source of power, is expended almost entirely in the charge. Little is wasted on the refractory lining or outside the furnace, thereby making working conditions better and more efficient at the same time.

Many Ajax-Northrup furnaces are made to accept either ferrous or non-ferrous work . . . with impressive savings for both. With non-ferrous alloys savings of over $33.00 a ton are reported in reduced metal losses alone. And for ferrous work, recovery is reported as high as 100% for nickel and 99% for chromium.

Economy recommends it, progress demands it; induction melting is fundamental to modern precision foundry work. Write for Bulletin 27-B . . . Ajax Electrothermic Corp., Trenton 5, New Jersey.

Associated Companies: Ajax Electric Company—Ajax Electric Furnace Co.—Ajax Engineering Corp.



AJAX NORTHRUP SINCE 1916

INDUCTION HEATING-MELTING

we completely redesigned it . . .

now it's better than ever!

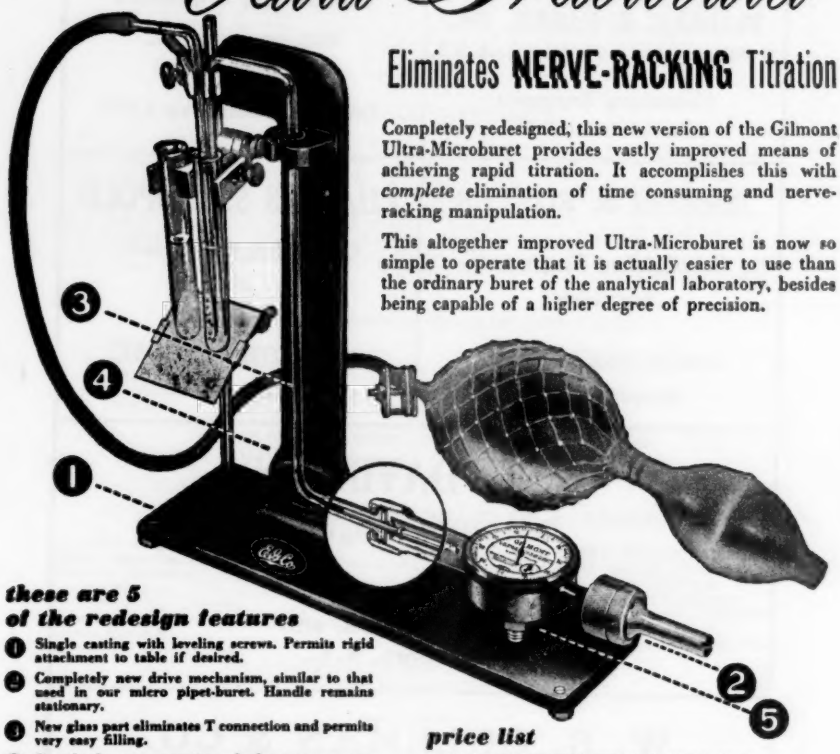
GILMONT

Ultra-Microburet

Eliminates **NERVE-RACKING** Titration

Completely redesigned, this new version of the Gilmont Ultra-Microburet provides vastly improved means of achieving rapid titration. It accomplishes this with *complete* elimination of time consuming and nerve-racking manipulation.

This altogether improved Ultra-Microburet is now so simple to operate that it is actually easier to use than the ordinary buret of the analytical laboratory, besides being capable of a higher degree of precision.



these are 5 of the redesign features

1. Single casting with leveling screws. Permits rigid attachment to table if desired.
2. Completely new drive mechanism, similar to that used in our micro pipet-buret. Handle remains stationary.
3. New glass part eliminates T connection and permits very easy filling.
4. Completely new mounting of glass part as employed in the micro pipet-buret.
5. Simple mounting of gauge reduces breakage of glass.

how the Microburet works*

A precision ground plunger, having the required diameter so that the dial micrometer gauge reads directly in volume units, displaces mercury in a reservoir sealed off by a silicone rubber gasket, contained in an aluminum bushing. The displaced mercury in turn forces exactly the same volume of titrating liquid through a fine orifice immersed in the solution being analyzed, which may be stirred by a fine stream of air bubbles.

*Gilmont, R., *Anal. Chem.* **20**, 1169 (1948)

price list

G 15391 A Ultra-Microburet, Gilmont, Capacity 1.0 ml. in .001 ml. divisions . . . . . Each $75.00

G 15391 B Same as G 15391 A but Capacity 0.1 ml. in .0001 ml. divisions . . . . . Each $75.00

G 15391 C Same as G 15391 A but Capacity .01 ml. in .00001 ml. divisions . . . . . Each $75.00

G 15392 Attachments (complete set including 3 clamps, test tube support, bubbling tube support, bubbling tube, milk glass, rubber bulb and panchclamp) . . . . . Each $19.00

Accuracy: within 0.1% of capacity. Constructed of corrosion resistant materials. Delivered filled with mercury ready for operation with instructions.

The EMIL GREINER Co.

29-26 N. MOORE STREET E&G Co. Dept. 234, NEW YORK 13, N.Y.

Consulting Engineers

HARRIS-DECHANT ASSOCIATES
Fidelity-Philadelphia Trust Building
Philadelphia, Pa.

FREDERIC R. HARRIS, INC.
27 William Street New York, N.Y.
Consulting Engineers
Naval Architects

EGGLY-FURLOW ENGINEERS
Consulting Engineers
PHILADELPHIA 2, PA.
1500 Walnut Street
Telephone: PEnnypacker 5-1197

DAMON & FOSTER
Consulting Engineers
Surveyors
CHESTER PIKE & HIGH ST.
SHARON HILL, PA.

CHARLES S. LEOPOLD
Consulting Engineer
◆
213 SOUTH BROAD ST.
PHILA. 2, PA.

P. L. DAVIDSON
Consulting Engineer
◆
Philadelphia, Pa. and
Greensboro, N. C.

W. B. COLEMAN & CO.
Metallurgists - Chemists - Engineers
Spectrographic and Spectrophotometric Analysis
Chemical and Physical Testing—Microscopic Investigation
Huey and Salt Spray Corrosion Tests—Boiler Water Conditioning
9TH & RISING SUN AVE., PHILADELPHIA 40, PA.

THE FRANKLIN INSTITUTE

1955

OFFICERS

President	S. Wyman Rolph
Executive Vice-President	Henry B. Allen
Vice-Presidents	G. H. Clamer James H. Robins Morton Gibbons-Neff
Secretary	Henry B. Allen
Treasurer	Hamilton Vaughan Bail
Assistant Secretary	Hamilton Vaughan Bail
Assistant Secretary	John Frazer
Assistant Secretary	Cecil M. Waterbury
Assistant Treasurer	Cecil M. Waterbury

BOARD OF MANAGERS

Term expires in 1956	*Term expires in 1957*	*Term expires in 1958*
A. Felix duPont, Jr.	Edward G. Budd, Jr.	Henry B. Bryans
J. G. Richard Heckscher	W. Laurence LePage	Francis J. Chesterman
Clarence L. Jordan	Hiram S. Lukens	James Creese
Ralph Kelly	John S. Malick	Rupen Eksergian
Lionel F. Levy	Richard T. Nalle	W. H. Fulweiler
Charles Penrose	Charles S. Redding	Gaylord P. Harnwell
R. G. Rincliffe	John Russell, Jr.	Harold S. LeDuc
Philip H. Ward, Jr.	W. Maxwell Scott, Jr.	Emery W. Loomis

The following are, *ex-officio*, members of the Board of Managers:

President
Executive Vice-President
Vice-Presidents
Secretary
Treasurer
Chairman of the Committee on Science and the Arts
Chairman of the Library Committee

George Wharton Pepper, Honorary Member of the Board

COMMITTEES OF THE BOARD OF MANAGERS

Committee on Board Vacancies	Charles S. Redding, *Chairman*
Budget Committee	James H. Robins, *Chairman*
Executive Committee	G. H. Clamer, *Chairman*
Nuclear Reactor	R. G. Rincliffe, *Chairman*
Personnel Policy Committee	Charles S. Redding, *Chairman*
Vermilye Medal Committee	Ralph Kelly, *Chairman*
Hostess Committee	Mrs. Thomas D. Thacher, *Chairman*

The President and the Executive Vice-President are *ex-officio* members of all Committees of the Institute.

COMMITTEES OF THE INSTITUTE

Bartol Research Foundation Committee	G. H. Clamer, *Chairman*
Biochemical Research Foundation Committee	A. Felix duPont, Jr., *Chairman*
Committee on Science and the Arts	Ralph H. McClarren, *Chairman*
Endowment Committee	Morton Gibbons-Neff, *Chairmn*
Finance Committee	Brandon Barringer, *Chairman*
Library Committee	Joseph S. Hepburn, *Chairman*
Meetings Committee	Nevin E. Funk, *Chairman*
Membership Committee	Morton Gibbons-Neff, *Chairman*
Museum and Memorial Committee	I. Melville Stein, *Chairman*
Publications Committee	Charles B. Bazzoni, *Chairman*
Committee on Research	G. H. Clamer, *Chairman*

The President and the Executive Vice-President are *ex-officio* members of all Committees of the Institute.

Adjusting "Mr. Meticulous" to make experimental junction tetrode transistors—devices which may find important uses in the telephone system.

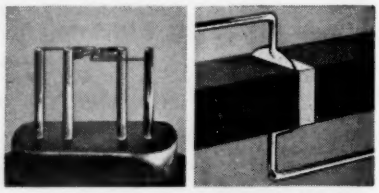
The machine we call "Mr. Meticulous"

Bell Laboratories scientists, who invented the junction transistor, have now created an automatic device which performs the intricate operations required for the laboratory production of experimental model transistors.

It takes a bar of germanium little thicker than a hair and tests its electrical characteristics. Then, in steps of 1/20,000 of an inch, it automatically moves a fine wire along the bar in search of an invisible layer of positive germanium to which the wire must be connected. This layer may be as thin as 1/10,000 of an inch!

When the machine finds the layer, it orders a surge of current which bonds the wire to the bar. Then it welds the wire's other end to a binding post. Afterward, it flips the bar over and does the same job with another wire on the opposite side!

Once only the most skilled technicians could do this work, and even their practiced hands became fatigued. This development demonstrates again how Bell Telephone Laboratories scientists work in every area of telephony to make service better.



Left, new transistor, magnified 6 times. Right, sketch of area where wires are bonded.

BELL TELEPHONE LABORATORIES

Improving telephone service for America provides careers for creative men in mechanical engineering.

Journal
of
The Franklin Institute

EDITOR, HENRY BUTLER ALLEN, MET.E., D.Sc., Sc.D., D.ENG.
ASSISTANT EDITOR, NANCY S. GLENN, M.A.

Associate Editors:

CHARLES B. BAZZONI, PH.D. — PAUL D. FOOTE, PH.D. — HENRY C. SHERMAN, SC.D.
ARTHUR L. DAY, SC.D. — C. E. K. MEES, D.SC. — W. F. G. SWANN, D.SC.
R. EKSERGIAN, PH.D. — WILLIAM B. MELDRUM, PH.D. — HUGH S. TAYLOR, D.SC.

Committee on Publications:

CHARLES B. BAZZONI, CHAIRMAN — GAYLORD P. HARNWELL — LIONEL F. LEVY
G. H. CLAMER — RICHARD HOWSON — HIRAM S. LUKENS
CLARENCE L. JORDAN

Vol. 260 **SEPTEMBER, 1955** **No. 3**

CONTENTS

How Commercial Titanium and Zirconium Were Born ..... W. J. KROLL 169

Matrices in Analogue Mathematical Machines ..... PIERRE M. HONNELL AND ROBERT E. HORN 193

Complex Zeros of the Error Function ..... HERBERT E. SALZER 209

On Electromagnetic Induction ..... PARRY MOON AND DOMINA EBERLE SPENCER 213

Nuclear Energy News 227

Notes from the National Bureau of Standards 230

The Franklin Institute

- Library 234
- Journal of The Franklin Institute 237
- Notes from the Bartol Research Foundation 238
- Notes from The Franklin Institute Museum 241
- Notes from The Franklin Institute Laboratories for Research and Development 243
- Notes from the Biochemical Research Foundation 245

Book Reviews 246

Publications Received 249

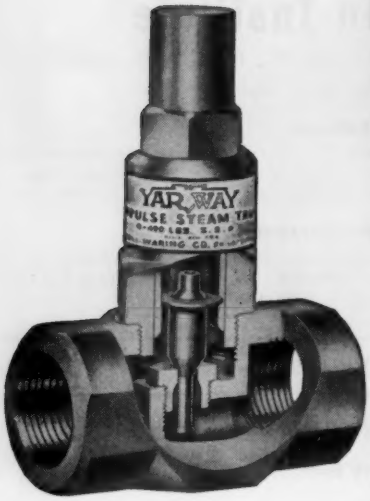
Current Topics 208, 212, 250

Published monthly by The Franklin Institute of the State of Pennsylvania, Benjamin Franklin Parkway at Twentieth St., Philadelphia 3, Pennsylvania. Printed by Lancaster Press, Inc., Prince and Lemon Sts., Lancaster, Pennsylvania.

Subscription rates: Domestic, $10.00 a year; Foreign, $11.00 a year. Single current issues, $1.00. Reprints, 50 cents per installment. All inquiries regarding subscriptions, change of address, sale of back issues and reprints, should be addressed to the Philadelphia office.

Indexes to the semi-annual volumes of the JOURNAL are published in the June and December numbers.

Copyright, 1955, by The Franklin Institute of the State of Pennsylvania.



YARWAY
impulse
steam trap

- Quick Heating
- Small Size
- One Moving Part
- Low Price
- Stainless Steel Construction

MORE THAN A MILLION SOLD

Known throughout industry as the steam trap that "gets equipment hot in a hurry and *keeps it hot.*" Free on request, Yarways 24-page booklet on steam traps and trapping problems. Ask for Bulletin T-1740.

YARNALL-WARING COMPANY
132 Mermaid Ave., PHILADELPHIA 18, PA.

A COMPLETE PRINTING SERVICE

GOOD PRINTING does not just happen; it is the result of careful planning. The knowledge of our craftsmen, who for many years have been handling details of composition, printing and binding, is at your disposal. For over seventy-five years we have been printers of scientific and technical journals, books, theses, dissertations and works in foreign languages. Consult us about your next job.

. PRINTERS OF .
JOURNAL OF THE
FRANKLIN INSTITUTE

LANCASTER PRESS, Inc.

PRINTERS • BINDERS • ELECTROTYPERS

ESTABLISHED 1877 LANCASTER, PA.

BIDDLE
Instruments

and miscellaneous laboratory equipment cover a wide field of electrical, mechanical and scientific testing, measuring and experimental work.

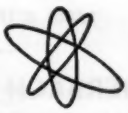
For a complete file of all our products write for Bulletin 19-53

JAMES G. BIDDLE CO.
Electrical & Scientific Instruments
1316 Arch Street—Philadelphia 7, Pa.

We specialize in supplying *your* needs . . .

From basic parts—such as resistors, tubes, condensers, and transformers—

To newer developments—such as vectors, turret assemblies, transistors, and germanium crystal diodes



FREE PARKING

ALBERT STEINBERG AND COMPANY
2520 N. Broad St., Philadelphia BA 3-9400
Greater Northeast 6324 Sackett St., Philadelphia JE 3-2994

Commercial Stationery
Loose Leaf—Blank Books
Filing Equipment
Office Supplies

SHANAHAN & CO.

22 S. 18 St. Ri 6-0333

WHERE QUALITY OF REPRODUCTION IS ESSENTIAL...
BUT ECONOMY IS IMPORTANT...

The answer is Atlas Photo Engraving Co.

1208 Cherry Street Philadelphia, Pennsylvania

BOLGER-PARKER COMPANY
Hauling and Rigging Contractors
752 N. MARKOE ST.
RESIDENCE PHONES
BOULEVARD 3295
SUNSET 9397M
PHILADELPHIA 39 PA.

Franklin Institute Books are bound by

FEHR & JOHNSON, INC.

Fine Bookbinding

831 Cherry Street
Philadelphia 7, Pa.

HEADQUARTERS
FOR
RADIO PARTS ELECTRONIC COMPONENTS
GEIGER-MÜLLER COUNTER TUBES
AND
RADIO ACTIVITY DETECTORS

HERBACH & RADEMAN
1204 ARCH STREET PHILADELPHIA 7, PA.

KEARNEY LUMBER COMPANY
Lumber of every description for every purpose
2916–26 Kensington Ave.
GA 6-5720 Phila., Pa.
"Our Fleet of Trucks Deliver Anywhere"

DISTRIBUTORS FOR WORTHINGTON
AIR CONDITIONING
COMMERCIAL REFRIGERATION
LOW TEMPERATURE SYSTEMS
ENGINEERING & REFRIGERATION, INC.
EV—6-3404 3325 CHESTNUT ST., PHILA.

EVERYTHING IN PAINTS
BUTEN'S
PAINT STORES
Phila., Chester, Reading, Upper Darby, Bryn Mawr, Darby, Doylestown
Camden, N. J. — Wilmington, Del.

PRECISION RULINGS ON GLASS
Scales · *Grids* · *Reticles*
Halftone Screens
MAX LEVY & CO. · Wayne Ave. & Berkley St. Phila., Pa.

Almo RADIO COMPANY
Executive Offices & Warehouse
412-16 N. SIXTH STREET • PHILADELPHIA 23 • PA.
Corner Sixth & Willow

8 LOCATIONS TO SERVE YOU

Executive Offices: 412 N. 6th Street, Phila. 23, Pa.

7540 FRANKFORD AVE. PHILA. 36, PA. De. 2-2234
509 ARCH ST. PHILA. 6, PA. Wa. 2-5153
6205 MARKET ST. PHILA. 39, PA. Gr. 6-1706
WILMINGTON, DEL. 1122 FRENCH ST. Wilm. 5-5161
ATLANTIC CITY 4401 VENTNOR AVE. At. City 2-5928
CAMDEN, N.J. 1133 HADDON AVE. Em. 5-1960
SALISBURY, MD. 317 PARK HEIGHTS Phone: 2-1393

ARTHUR H. THOMAS COMPANY
Laboratory Apparatus and Reagents
Selected for Chemistry and Biology
Adequate stocks of 16,000 Apparatus items and 6,000 Reagent items carried in our Warehouse stock for immediate shipment
WEST WASHINGTON SQUARE PHILADELPHIA 5, PA.

HESS & BARKER
ENGINEERING MACHINISTS
212 S. DARIEN ST.
PHILA., PA.

AWARDS BY THE INSTITUTE

The Franklin Medal (1914—Gold Medal).—This medal is awarded annually to those workers in physical science or technology, without regard to country, whose efforts, in the opinion of the Institute, acting through its Committee on Science and the Arts, have done most to advance a knowledge of physical science or its applications.

The Elliott Cresson Medal (1848—Gold Medal).—This medal is awarded for discovery or original research, adding to the sum of human knowledge, irrespective of commercial value; leading and practical utilizations of discovery; and invention, methods or products embodying substantial elements of leadership in their respective classes, or unusual skill or perfection in workmanship.

The Howard N. Potts Medal (1906—Gold Medal).—This medal is awarded for distinguished work in science or the arts; important development of previous basic discoveries; inventions or products of superior excellence or utilizing important principles.

The John Price Wetherill Medal (1925—Silver Medal).—This medal is awarded for discovery or invention in the physical sciences or for new and important combinations of principles or methods already known.

The Edward Longstreth Medal (1890—Silver Medal).—This medal is awarded for inventions of high order and for particularly meritorious improvements and developments in machines and mechanical processes. In the event of an accumulation of the fund for medals beyond the sum of one hundred dollars, it is competent for the Committee on Science and the Arts to offer from such surplus a money premium for some special work on any mechanical or scientific subject that is considered of sufficient importance.

The Louis E. Levy Medal (1923—Gold Medal).—This medal is awarded to the author of a paper of especial merit, published in the JOURNAL OF THE FRANKLIN INSTITUTE, preference being given to one describing the author's experimental and theoretical researches in a subject of fundamental importance.

The George R. Henderson Medal (1924—Gold Medal).—This medal is awarded for meritorious inventions or discoveries in the field of Railway Engineering.

The Walton Clark Medal (1926—Gold Medal).—This medal is awarded to the "author of the most notable advance in knowledge or improvement in apparatus, or in method concerning the science or the art of gas manufacture or distribution or utilization in the production of illumination, or of heat, or of power."

The Frank P. Brown Medal (1938—Silver Medal).—This medal is awarded to inventors for discoveries and inventions involving meritorious improvements in the building and allied industries.

The Newcomen Medal (1943—Gold Medal).—This medal is awarded, not oftener than once in three years, for achievement in the field of Steam.

The Francis J. Clamer Medal (1943—Silver Medal).—This medal is awarded at least once in five years for meritorious achievement in the field of Metallurgy.

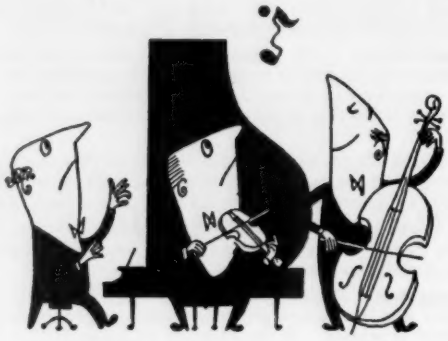
The Stuart Ballantine Medal (1946—Gold Medal).—This medal is to be awarded in recognition of outstanding achievement in the fields of Communication and Reconnaissance which employ electromagnetic radiation.

The Boyden Premium (1859).—This premium is awarded not oftener than once in five years to any resident of North America who has recently made a notable experimental determination of the speed, in free space, of radiation in any region of the entire spectrum.

The William M. Vermilye Medal (1937—Bronze Medal).—This medal is awarded not oftener than biennially in recognition of outstanding contribution in the field of Industrial Management.

The Certificate of Merit (1882).—A Certificate of Merit is awarded to persons adjudged worthy thereof for meritorious inventions, discoveries or improvements in physical processes or devices.

For further information relating to these awards apply to The Executive Vice-President



SOUR NOTE?

Today the job of managing an estate requires *harmony* among specialists in administration, taxation, investments and accounting. One sour note and the whole plan can go wrong. That's why you and your attorney will be wise to depend on the *group action* of our Trust Department.

FIDELITY-PHILADELPHIA TRUST COMPANY

Bankers since 1866

Broad and Walnut Streets
Philadelphia 9

"A billion-dollar trust company—a great commercial bank"

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 260 SEPTEMBER, 1955 No. 3

HOW COMMERCIAL TITANIUM AND ZIRCONIUM WERE BORN

BY

W. J. KROLL[1]

INTRODUCTION

It has been claimed that the arrival of commercial titanium is the greatest event in nonferrous metallurgy since the introduction of aluminum electrolysis by Hall-Heroult. Present investments in titanium industries may exceed three hundred million dollars, and with the ultimate production goal of two hundred thousand tons of titanium per annum, the capital spent on plants will, within the next ten years, go into the billions. Judging from the numerous articles that have appeared in the daily press, titanium—the "wonder metal"—has captivated the imagination of the people. Legitimately the question is being asked why this metal, after so many years of neglect, made this crashing break through and who the men are who brought this all about.

Dr. Henry B. Allen, Vice President of The Franklin Institute, during the ceremony at which the Francis J. Clamer medal was awarded to me,[2] asked me about my experiences in the development of titanium. It is with some hesitation that I write down at his request the gist of our ensuing conversation, knowing too well how much I owe my predecessors who erected the foundations of titanium metallurgy. Also, while the production method which I favor is still used predominantly today, no one knows what process will emerge a few years from now as the better one, with the intensive research work done on this subject all over the world. Maybe my contribution then, in the perspective of time, will be considered, if at all, only as trivial. I am nevertheless aware of the fact that I started the titanium avalanche by loosening the little bit of snow that got it under way on its thundering

[1] Consulting Metallurgist, Corvallis, Oregon.

[2] At Medal Day, October 20, 1954, at The Franklin Institute in Philadelphia, Pa.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

road. This may be reason enough to tell my part of the story before the curtain falls before me, the more so since many of my predecessors are still alive, who may confirm or refute my assertions. This contribution to history also will eliminate any mythology that usually surrounds events of this kind.

EARLY LIFE

In any invention of importance one has first to examine the educational background of the inventor and his mental preparation and development. Besides this, there are also time elements that cannot be neglected for successful industrialization of an invention. This leads me to describe the early years of my life and the evolution through which I went, often deeply influenced by my ambiance.

The fact that I was born into a family with a tradition in metallurgy—my father having been a manager of a blast furnace plant, one of my grandfathers having been operator of an iron ore mine—contributed certainly to my mental conditioning. A far away picture still remains in my mind, when, as a child, in the darkness of a cold morning I was taken out of bed to light a new blast furnace with a bunch of cotton, impregnated with burning kerosene and fastened to the tip of a long pole. My high school education in the walled, medieval capital of Luxembourg was uneventful but not without influence. Greek and Latin left their mark on me as well as the liberal internationalist spirit of that democratic little country that succeeded for centuries in playing ball with big neighbors, without losing its independence. The laboratories of the Technische Hochschule in Charlottenburg, Germany, where I studied iron metallurgy from 1910 to 1917, were well equipped with all sorts of furnaces. I spent the years of the First World War there as an assistant to the professor of iron metallurgy, W. Mathesius, and worked on my doctoral thesis under K. A. Hofman on the production of pure boron (**1**).[3] This influenced my future life deeply, for I never strayed far from the chosen field of preparative chemistry.

LEAD REFINING

I was hired at the end of the war by the Metallgesellschaft in Frankfurt/Main, and sent to their lead refinery at Call/Eifel, in the mountains. There I put to practice an idea of mine, to produce calcium lead to be used as bearing alloy and as a substitute for scarce tin Babbitt metal. I tried first to react calcium carbide with lead (**2**), which worked fine in the laboratory, but I did not succeed on an industrial scale because of the high temperature involved when working in the absence of fluxes.

Then I decided to react fused sodium lead with calcium chloride (**3**) but, to lower the melting point of the salts, I used a mixture of calcium/

[3] The boldface numbers in parentheses refer to the references appended to this paper.

barium chloride and obtained a calcium/barium lead alloy, which was later sold in large quantities for bearings under the name "Lurgimetall". A similar alloy "Frary Metal" was produced at about the same time quite tediously by fusion electrolysis, in this country. While my process permitted making the alloy in a few hours, the fusion electrolysis took almost a week (51). However, my idea was not new, and Caron (52) had used it fifty years before, as I discovered to my sorrow by trying to get a patent on it. The production of this alloy on a larger scale was not easy at the beginning since it involved the use of up to 120 kilos of sodium, which was injected in liquid form by CO_2 pressure in a few minutes in a large lead bath, covered with the fused salts. (See Fig. 1.) Sodium in those days was produced by the fusion

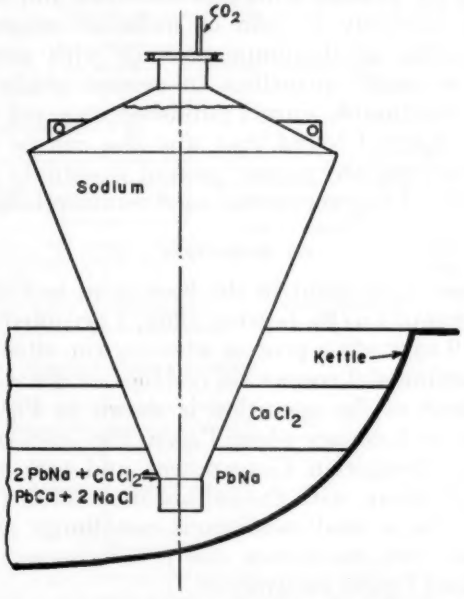


FIG. 1. Sodium injector for producing calcium lead.

electrolysis of NaOH and it sometimes contained a lot of hydride. This, in contact with lead, gave off all its hydrogen and the fires I had to combat in the first large scale operation would have made me fit as a fire fighter in the Second World War's London Blitz.

The Ca/Ba bearing alloy made was rather pasty because of the presence of the compound Pb_3Ca, which liquated out on cooling, and I wanted to produce a more fluid alloy. For this purpose I made additions of antimony or bismuth, but I found that the alloy became hereby worse. I analyzed the liquated crust and found it to contain practically all of the Sb or Bi. Therefore I concluded that I had there

a fine process for debismuthizing lead—which method I patented (4, 5). Bismuth is very harmful in lead. It stains white lead used as a pigment and negative storage battery plates distort when containing bismuth, on rapid discharging. I put my debismuthizing process to practice later, in 1921, on a five ton charge in the Norddeutsche Affinerie, Hamburg and lowered the bismuth content from 0.3 to less than 0.05 per cent by adding calcium lead and skimming off the dross. I found that alkali metals worked similarly but not as efficiently.

Then I discovered that aluminum can be used to deantimonize and dearsenize tin alloys, especially solder (6). This process was put to practice by Th. Goldschmidt Co. Essen, to which I sold this idea. Grievous accidents happened with the aluminum dross, which some companies, using my process in foreign countries and especially in the U.S.A., exposed carelessly to rain or moisture whereby arsine was developed by reaction of aluminum arsenide with water. Arsenic is usually present in small quantities in second grade solders. This carelessness was inexcusable since I published repeated warnings about this hazard (7). Later I found that zinc also can be used to deantimonize solder providing any copper present is entirely removed before the operation (50). This process was used commercially for some time.

TIN EXTRACTION

After about one year spent in the lead plant in Call, to get under way the production of Ca/Ba bearing alloy, I accepted a mission to go to Vienna in 1919 to study a process whereby tin, silver and gold were extracted from antimonial copper tin residues, originating from church bells. A flow sheet of the operation is shown in Fig. 2. With this knowledge I went to Hungary where I spent the years of 1920 and 1921 on behalf of the Hungarian Government and two other associated companies to build there, with the help of the staff of the big Manfred Weiss, Csepel works, a small nonferrous metallurgy plant comprising one shaft furnace, two converters, one reverberatory furnace besides the existent tin and copper electrolysis.

The start of this plant was nasty. The first night of operation of the tin reverberatory furnace I was called out of bed to look over the baghouse. Every one of the 7-meter long woolen bags could be torn apart with the finger tips—sulphuric acid had ruined them all in a few hours. The checkerwork of the regenerators of the gas heated reverberatory furnace made sulphuric acid of the sulphur contained in the waste gases by a neat catalytic action of SnO_2 on SO_2 and damp, oxygen bearing combustion gases. There was nobody to help me. My cables to the sponsoring companies remained unanswered. The situation looked desperate since there was no easy means known how to eliminate the high sulphur content of the gas producer gas, used for the heating of the reverberatory, nor of removal from the sulphur-

rich batch. I had an idea at the right moment. I sent to a brewery for an ammonia cylinder, and told the baghouse attendant to apply to each chamber, after shutting it down and shaking the bags, enough ammonia until it started smelling. This was a success since we could immediately resume our production after refitting the baghouse. This process, which I tried to patent, was not new, however, as it had been used before by the Beth Filter Company in Saarbrucken, Germany.

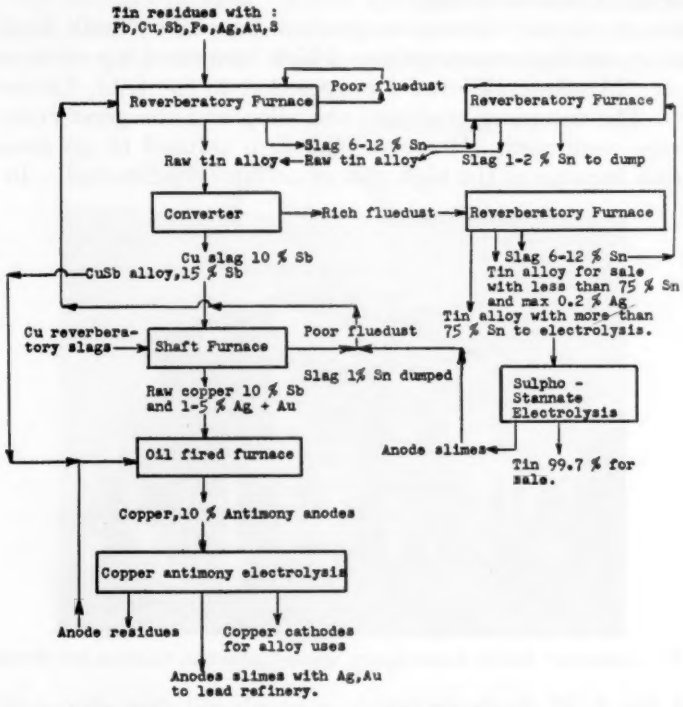

Tin residues with : Pb,Cu,Sb,Fe,Ag,Au,S
Reverberatory Furnace
Poor fluedust
Reverberatory Furnace
Slag 6-12 % Sn
Raw tin alloy
Raw tin alloy
Slag 1-2 % Sn to dump
Converter
Rich fluedust
Reverberatory Furnace
Cu slag 10 % Sb
CuSb alloy,15 % Sb
Slag 6-12 % Sn
Tin alloy for sale with less than 75 % Sn and max 0.2 % Ag.
Tin alloy with more than 75 % Sn to electrolysis.
Cu reverbera-tory slags
Shaft Furnace
Poor fluedust
Slag 1% Sn dumped
Sulpho - Stannate Electrolysis
Raw copper 10 % Sb and 1-5 % Ag + Au
Anode slimes
Tin 99.7 % for sale.
Oil fired furnace
Copper,10 % Antimony anodes
Copper antimony electrolysis
Anode residues
Copper cathodes for alloy uses
Anodes slimes with Ag,Au to lead refinery.

FIG. 2. Flow sheet of the process used for reclaiming tin, copper and noble metals from church bell residues.

It was, of course, impossible to neutralize the hundreds of pounds of sulphuric acid that passed through the filter fabric every day but it was easy, with but little ammonia, to take care of the small quantity of sulphuric acid that had condensed on the woolen fiber between two ammonia treatments, before it caused trouble.

ALUMINUM ALLOYS

After leaving Hungary in 1922, the tin job having been accomplished successfully, I associated for a short time with a small foundry in Baden-Baden, Germany, where I set up the rudiments of a research laboratory. The first experiments in this place were quite successful.

They concerned the creation of a low expansion piston alloy with good wear resistance, good creep strength and light weight. I developed there the hyper-eutectic 23% Si-Al alloys with small additions of copper, sold later for many years as "Alusil" in Germany and as "Alsia" in France (**8**). Favorable circumstances permitted the production of this alloy since it was brought out just at the right moment when sintered carbide tools made possible the machining of grooves in pistons cast of this hard alloy.

I also developed then a magnesium-base alloy with high creep strength at medium temperatures which contained up to 6 per cent cerium or "Mischmetall" and I proposed it to the I. G. Farbenindustrie (**9**). The company produced the alloy and the good creep properties were confirmed. However, the firm refused to go ahead with production because of the high cost of cerium-Mischmetall. In World

FIG. 3. Laboratory Bel'air, Luxembourg, where commercial titanium was developed.

War II the I. G. Farbenindustrie remembered this alloy and it was put in superchargers of aviation motors. There it was discovered by American metallurgists in shot down planes and copied (**53**). It still is one of the outstanding creep-resisting light metal alloys used in aviation.

After liquidating my German interests in 1923 I decided to start a research laboratory of my own in my home town of Luxembourg where I bought a large house and transformed it for my purposes. I operated this laboratory with the help of one unskilled laborer, one mechanic and a secretary who also did some metallographic and spectographic work. It is shown in Fig. 3. There I spent all my time up to February 1940 when I fled to the U.S.A. before the German invasion, not wanting to work for Hitler, leaving behind my property with all the expensive equipment, bought over many years.

DEBISMUTHIZING PROCESS

Soon after establishing myself in Luxembourg, the debismuthizing process mentioned above found considerable development in the U.S.A., where Mr. Jesse O. Betterton of the American Smelting and Refining Co. had put it to practice on a big scale without my knowledge (**54**). While the tolerance for bismuth in lead was only 0.001 per cent in Europe, in the U.S.A. one accepted 0.05 per cent, which was also the limit of my calcium process. These specifications made my method inoperable in the Old Country but suitable in the U.S.A. But Betterton improved on my original idea by using magnesium in addition to

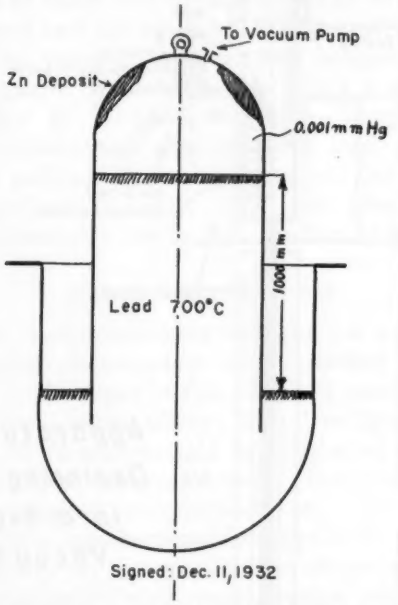


FIG. 4. Vacuum dezincing of Parkes lead in a submerged bell jar with lead seal.

calcium, whereby he could lower the final bismuth content to less than 0.02 per cent and, with small additions of antimony, he could even go below the 0.001 per cent mark.

In this period I tried to make my above-mentioned carbide process behave for the production of calcium lead. I reacted CaC_2 with fused lead under a flux of $NaCl/CaCl_2$ in a protective atmosphere or under kerosene gas (**10**). By dipping the carbide under the salts and into the lead with a scoop I readily obtained calcium-sodium-lead alloys of any wanted composition, also low in sodium. My trend of ideas of making calcium lead from carbide in presence of a flux had been followed independently by Betterton (**55**), who used a mixer to bring about a contact of the reagents with lead. After becoming aware

of my U. S. patents, the American Smelting bought my debismuthizing rights. The Kroll-Betterton debismuthizing process is applied today to about ten per cent of the world production of lead as far as it is bismuth bearing. It eliminates the necessity of setting up an expensive Betts electrolysis.

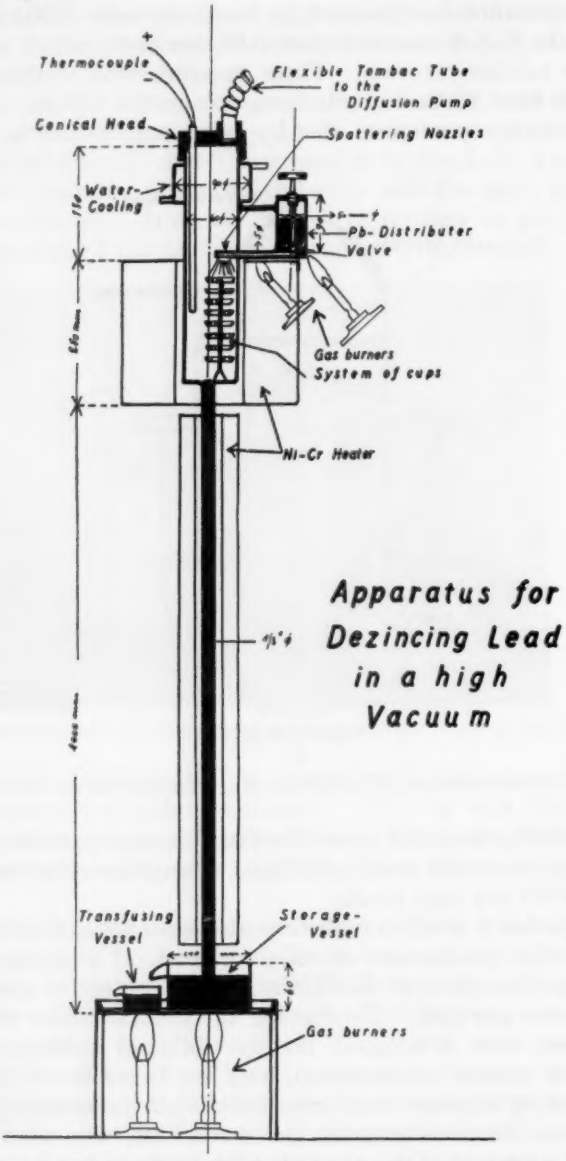


FIG. 5. Vacuum dezincing of Parkes lead with circulation and lead sprays.

My connection with the American Smelting and Refining Co. brought about some research work in other lines of lead refining. On December 11, 1932 I submitted to the company a process for dezincing lead of the Parkes desilverization process in a vacuum by the arrangement shown in Fig. 4 as taken from the preserved original drawing. I wrote about this possibility in 1935 and stressed the necessity of "renewing the surfaces" (11). On January 25th, 1935 I disclosed to the aforesaid company further results of my laboratory dezincing experiments in which the equipment of Fig. 5 was shown. As seen herefrom all the elements of present day vacuum dezincing of lead are contained in these two drawings and in my publication, especially the submerged bell jar and the agitation of the bath. But the company showed no interest, nor did the Belgian Company Société Génerale Metallurgique de Hoboken to which I also submitted this idea. However, Mr. W. T. Isbell of the St. Joseph Lead Company, whom I had told about this possibility of lead dezincing and who certainly knew my publication, went ahead and developed the vacuum dezincing process industrially (73). The basic idea of vacuum dezincing of lead is not patentable nor is the agitation of the bath on account of previous art.

BERYLLIUM AND GERMANIUM

The early work in Luxembourg brought me to examine the duralumin age-hardening phenomenon and I looked for substitutes for silicon, magnesium and copper in this alloy. I soon discovered the age hardening of aluminum silver alloys (12). I recognized that the aging in lithium-bearing Al-Zn alloys could be attributed only to the precipitation of a lithium zinc compound (13). Beryllium did not replace either silicon or magnesium in duralumin (14). Germanium acted just like silicon and gave age-hardenable Mg-Ge-Al alloys (15). This brought about a study on the equilibrium diagram Ge–Al and I discovered that germanium crystals could replace silicon in radio detectors (15), which was the predecessor of present-day transistors. The investigation on beryllium engaged me to build a fusion electrolysis cell for the production of the metal by the Stock-Goldschmidt process, using the double fluoride as an electrolyte. I produced fused beryllium all right but became beryllium poisoned as could be expected.

My work on the beryllium electrolysis naturally brought me to examine better methods of production. In October 1927 I reported to Siemens & Halske about the reduction of anhydrous beryllium fluoride with magnesium—a method I later patented (22). Beryllium metal was rather uninteresting at that time and so, until its use in atomic piles became apparent, this method was not put to practice until 1944, when the Brush Beryllium Co. applied it with unessential modifications on a commercial scale (59). This is today the only process of beryllium production in the U.S.A. I used the same procedure

for producing by reduction of BeF_2 with Mg in presence of Cu, Fe or Ni powders or turnings under argon, alloys of any desired composition.

My publication on beryllium electrolysis (16) brought me in contact with the German firm Siemens & Halske which tried to develop the Stock-Goldschmidt method of beryllium production. We agreed to exchange knowledge. Later these relations brought about a contract for more intimate collaboration in the general field of rare metals. The company paid me a small retainer to keep an assistant but the bulk of the expense fell on me. I hoped to be refunded some day by the sale of patent rights.

VACUUM METALLURGY

My interest in vacuum metallurgy started the day when the Heraeus Vacuumschmelze, Hanau was taken over by Siemens & Halske. Dr. W. Rohn, the great pioneer of vacuum metallurgy, was at that time manager of the Vacuumschmelze. He produced gas-free metals on a big scale, such as nichrome for thermocouples, nickel for electronics, oxygen-free copper, and especially large (up to 2-ton) batches of degassed steel (56). I had, however, to learn my vacuum techniques by myself, since I was not admitted to the Heraeus plants until many years later. A good introduction to vacuum techniques was brought about by a job I received from Siemens & Halske, namely to produce a few kilos of barium, and to fill it into thin nickel tubes for getters. I used the Guntz process of vacuum reduction of BaO with aluminum (17, 18), made many kilos of metal but had my troubles in filling it in the nickel sleeves. After that I decided to produce larger quantities of high purity calcium from the electrolytic grade by subliming *in vacuo*. This was achieved with relative ease and the heavy calcium ingots were cast from the condensate under argon (19). As can be seen, this gave me the background of knowledge for developing a titanium and zirconium metallurgy in which vacuum and noble gases play such an important role.

Naturally I looked for applications for the high purity calcium thus obtained. I put it to use for oxide reductions in a bomb, producing in this way hot malleable fused granules of chromium and vanadium in addition to powders of titanium, zirconium, uranium and thorium. All these metals, after sintering or melting, were found to be hot malleable (20). The two latter were also ductile as cold. In the case of chromium, I also substituted the chlorides for the oxide. Such methods which were already known at that time have become quite popular at present in the U.S.A. I furthermore applied my knowledge to the vacuum purification of metals and alloys, especially silicon, copper, iron, chromium, beryllium and the alloys of copper, tin and lead (11). An order to distill one pound of beryllium in a vacuum using self-made beryllia crucibles and condensors was successfully accomplished.

TITANIUM

My first notes on titanium go back to September, 1930, when I reduced $TiCl_4$ with sodium by the Nilson & Petterson method (57), called the Hunter process in the United States (58). A heavy wall steel bomb was filled with clean sodium lumps, which were submerged in pure titanium chloride. After bolting down the top on the gasket made of soft electrolytic iron, the bomb was heated up until a flash occurred. One operation gave 386 grams of titanium granules with a recovery of 58 per cent which shows the scale of the experiment. Some bombs blew, which discouraged me from using this method later on. I succeeded in rolling down a large fused pellet, as produced by this process, to sheet and showed it in 1932 on a trip to the U.S.A. to the American Smelting and Refining Co., to the Bell Telephone Co., to the Westinghouse-Bloomfield Co. and to the General Electric Co. in Schenectady without exciting any interest. I used such metal later for the development of age-hardenable copper titanium alloys (21) and to establish part of the equilibrium diagram.

Being convinced that a flash reaction creating high pressure (as is the one between $TiCl_4$ (B.P.136°C.) and admixed sodium (B.P. 880°C.)) would never become commercial I decided to investigate the reducibility of TiO_2 with pure calcium obtained by vacuum sublimation. For this purpose an internally heated reactor was set up, which was operated under argon (23). The first runs took place according to notes in May 1935. At that time I also reduced zirconium oxide in the same way. The products, after acid extraction, pressing and vacuum sintering, could be hot rolled to sheet but were brittle when cold. Double reduction with calcium hydride under argon for the second step did not improve the metal. Present day practice confirms this fact.

The first experiments with the pressureless reduction of $TiCl_4$, with calcium under argon, took place on June 17, 1937. A series of 18 runs was accomplished, each with an output of about 250 grams titanium showing a yield of 88 per cent. The best hardness values obtained at that time were around Brinell 200. The metal was cold malleable.

The idea of reducing without a pressure buildup was a logical conclusion made after mixtures of sodium and titanium chloride, flashed in a bomb, had blown up. It was thought that the reagents should be admitted only as needed, to avoid explosion. Hot titanium is spoiled by air. A vacuum in the reactor was not considered as practical because of the volatility of the reducing agent at elevated temperature, which would have led to accretions above the bath level. This left only argon or a noble gas as the means for creating the substantially atmospheric pressure wanted for safe operation, especially in a laboratory. The argon was soon found to offer an excellent means for control of the reaction velocity if all the reducing agent was admitted at the start of the run, because, the more noble gas is present, the

slower the reduction of $TiCl_4$ proceeds. Also a runaway could be cut down by admitting argon. A rubber balloon was used as a convenient means to balance the pressure inside the reactor against the atmosphere.

The first reductions with calcium took place in an iron crucible, held in a quartz tube filled with argon, heated by high frequency induction, as shown in Figs. 6 and 7. A glass funnel permitted drip-

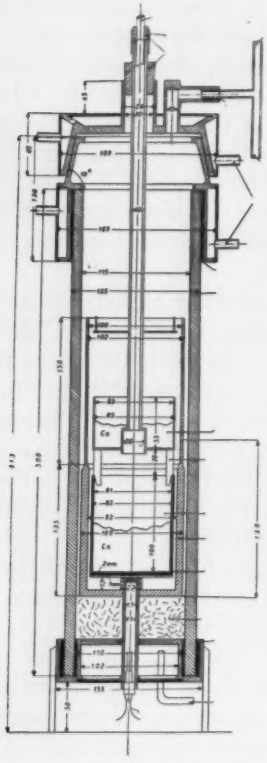


FIG. 6. High frequency quartz tube furnace for the reduction of $TiCl_4$ with fused Ca or Mg under argon.

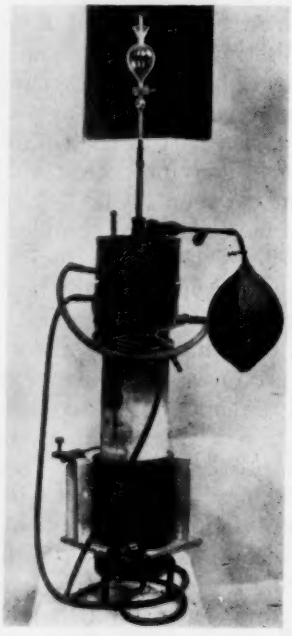


FIG. 7. External view of high frequency quartz tube furnace for titanium reduction, shown in Fig. 6.

ping $TiCl_4$ on the fused reagent. A number of development drawings have been preserved. A first report on this process was sent to the Siemens & Halske Co. under the date of August 27, 1937. On July 30th, 1937 I switched from calcium to magnesium as a reducing agent, using the same equipment. The hardness of the metal was higher with magnesium, usually above 230 Brinell, but later, with a larger, externally heated stainless steel vessel, batches of up to 4 lb. could be obtained with a Brinell hardness down to 180 (**24**, **25**, **26**) (Fig. 8). A surprising

difficulty arose one day when the metal produced became unexpectedly hard. This happened after omitting the HCl pickling of the rough titanium powder, as produced from the turnings by pounding in a steel mortar under water. It was found that the pounding of the turnings creates enough local heat at the point of impact to cause a

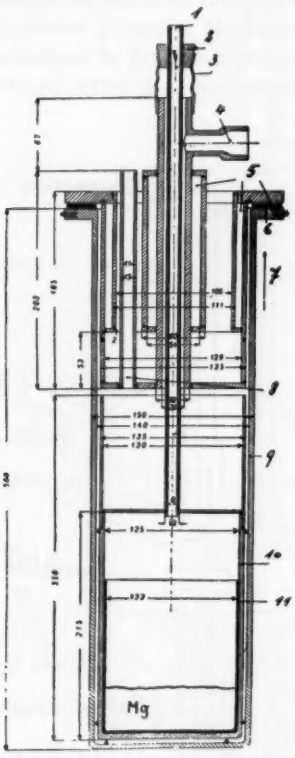


FIG. 8. Titanium reactor.

1. inlet for titanium tetrachloride
2. rubber plug
3. rubber tire
4. connection to the vacuum pump
5. water cooling
6. rubber gasket
7. cooling by means of a lead coil
8. opening for a getter electrode
9. nichrome container
10. iron crucible
11. molybdenum lining

reaction of the titanium with water, and local oxidation. The acid leaching removes this oxide, at least to a large extent. This aqueous method of salt removal was later adopted by F. S. Wartman (60). On July 13, 1938 the first zirconium reductions were made with a similar equipment, provided with a $ZrCl_4$ evaporator, Also, vacuum separation of the sponge from the chloride was then put to use. This is the

basic conception of the present-day zirconium process, as developed industrially in the Albany, Oregon station of the U. S. Bureau of Mines.

The sponge, as obtained after salt removal, was in the beginning vacuum sintered but later arc melted. The idea of arc melting had already been used by Werner von Bolton in 1905 (61) for the melting of tantalum. In 1932, I saw this process in operation at the Siemens & Halske plants in Siemensstadt where a series of arc melting furnaces was set up. These were constituted of individual large glass bulbs as shown in Fig. 9. The tantalum briquette, made of sheet clippings, was

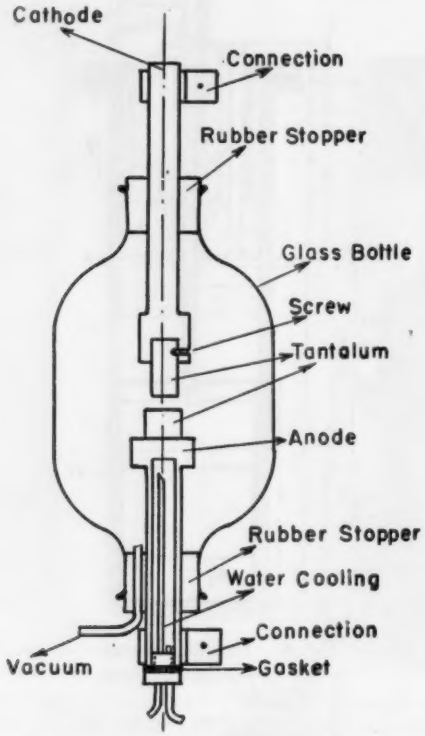


FIG. 9. Von Bolton's vacuum-arc furnace.

melted down on a water-cooled copper plug with a tantalum electrode, which usually was contaminated with some oxide to make it less fusible. A slight pressure of hydrogen was maintained in the system to avoid a glow discharge. A crude mechanical pump created the vacuum for a series of furnaces connected in parallel with the main line by way of stopcocks. Direct current was used and the tantalum was the anode. The buttons so produced were very ductile and they could be rolled down to sheet without annealing, when cold. I found this

furnace arrangement too crude for titanium and switched to an all metal construction. (See Fig. 10.) I used a diffusion pump for the evacuation and backfilled with pure gettered argon of a pressure high

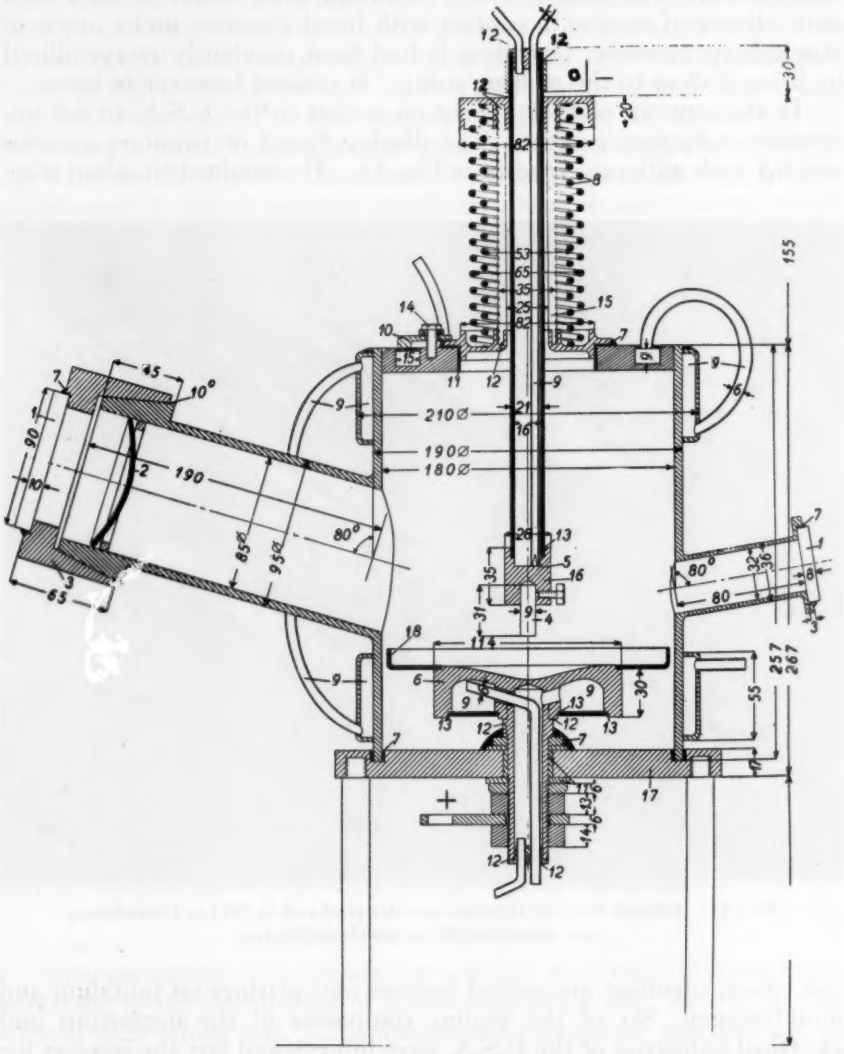


FIG. 10. Vacuum-arc furnace.

enough to avoid a glow discharge. A thoriated tungsten electrode tip was used for the melting (25).

Arc melting had become necessary for titanium after I carefully studied the behavior of various refractories in contact with the fused

metal which showed that there was a reaction by which oxygen was given off. The following oxides were found to react with fused titanium: BeO, Al_2O_3, MgO, CaO, ThO_2, ZrO_2 and La_2O_3 (**25, 26**). The best behavior was that of ThO_2. Calcium oxide could be used with some chance of success in contact with fused titanium under argon of atmospheric pressure, providing it had been previously recrystallized by firing it close to the melting point. It reacted however *in vacuo*.

In the autumn of 1938 I went on a visit to the U.S.A. to sell my titanium-reduction process. The display board of titanium samples which I took with me is shown in Fig. 11. It contained titanium wire,

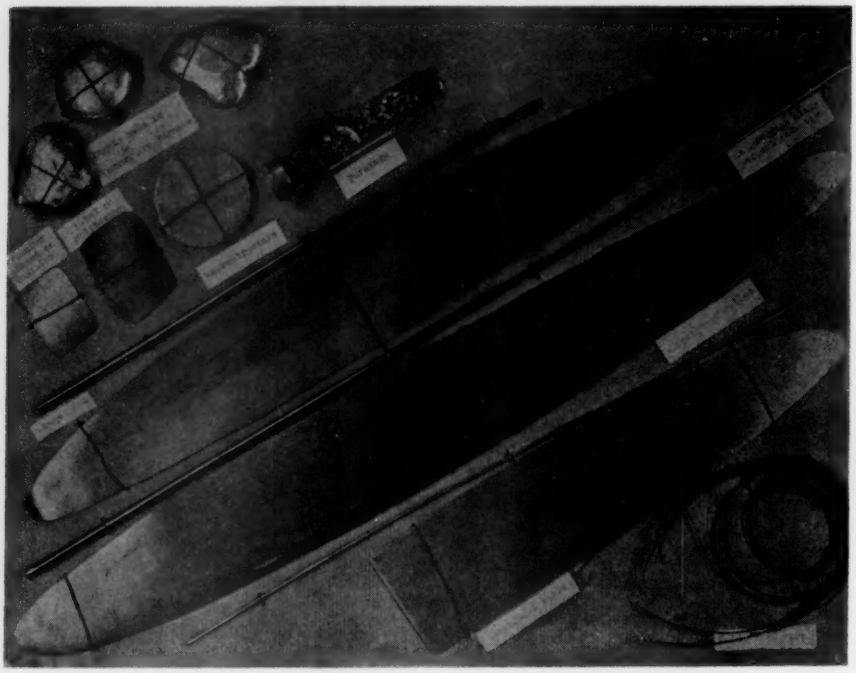


FIG. 11. Display board of titanium samples produced in 1937 in Luxembourg and shown in 1938 in the United States.

rod, sheet, turnings, arc-melted buttons and platings on tantalum and molybdenum. Six of the leading companies of the nonferrous and electrical industries of the U.S.A. were interviewed but the interest for titanium was nil, despite the fact that I stressed the availability of the ores, the good corrosion properties of the metal, the mechanical strength comparable with that of stainless steels. I left the United States in a sad state of mind, not having been able to interest anybody in my ideas.

AGE-HARDENING

While working on my main projects I was frequently interrupted to examine sidelines. Among these there was an investigation of age-hardenable iron-beryllium alloys and the possible substitution of beryllium in such alloys in general. The first resulted in the discovery that an addition of nickel to beryllium steels permits large savings of beryllium for a given hardness and besides this considerable grain refining was obtained (**27**). A substitution of titanium or aluminum for beryllium in nickel-bearing steels was also found to impart considerable age-hardenability (**28**). Chromium could be present and hardenable stainless steels with titanium or aluminum additions were thus produced. Such alloys have acquired importance in recent days because of their high yield strength and good mechanical properties at medium temperature where they might well compete with titanium.

The age-hardening of titanium and aluminum nickel steels is today commonly used for improving magnetic properties in the so-called Mishima magnet steels. I could not investigate the magnetic properties at the time I discovered the precipitation hardening of these materials, lacking the necessary equipment, but urged checking them, without success, on the German electricity trust for which I was working. This was one of the many buses which I missed.

Looking for a substitute for beryllium in age-hardenable nickel-base alloys, I discovered one day a considerable precipitation hardening with commercial nickel to which I had added 0.2 per cent magnesium for deoxidation. The analysis revealed that there was also 0.2 per cent carbon present and that the hardening effect could be obtained only by the combination of magnesium and carbon (**29, 30**). This property extends also to some other high nickel alloys as for instance Monel metal. I disclosed this fact in 1932 to the International Nickel Company, which later used this alloy for many years and sold it under the name Z-Nickel for corrosion-free springs to be used especially in fine machinery (**31**). A license agreement was ready in 1940 but the outbreak of war and seizure by the Alien Property Custodian put an end to this attempt at legalizing an infringement situation.

PURIFICATION OF METALS

Vacuum distillation was another subject of various investigations which led to the purification of metals such as Cr, Mn, Be, Fe, Sn, Zn, Pb and their alloys (**11**). Aluminothermic manganese was distilled by the pound and the distillate was used for various purposes. I found that manganese, if pure enough, could readily be rolled above 1100° C., providing it was not cooled down below this temperature after melting in alumina refractories (**32**). A publication by Grube on palladium manganese alloys (**62**) showed that with a certain addition of this element manganese became soft and I thought that there should be

some cheaper additions that would permit retaining the high temperature ductile gamma manganese form at ambiance. A short investigation proved that copper and nickel both stabilize the ductile form (32, 33). Additions of aluminum increased the quality of castings made from these alloys. Further experiments disclosed at once that the new alloys had a very high coefficient of expansion and high electric resistivity. Similar investigations were underway at the same time without my knowing at the U. S. Bureau of Mines, and Dr. R. S. Dean later described the discoveries made, which confirmed my own (63). However, I missed the damping properties of these alloys.

WORK IN UNITED STATES

Under the menace of World War II I left Luxembourg on February 10, 1940 for New York on an immigration visa, two months ahead of the invasion of that country by Hitler. I abandoned all my belongings, the whole equipment of my laboratory and the big estate. Luck was with me since I got it back again in 1947, almost intact. In October 1940 I presented the result of my titanium research before the American Electrochemical Society in Ottawa (25).

From 1940 to 1945 I stayed with the Union Carbide Research Laboratories in Niagara Falls, as a consultant. This was a fertile time. It brought about investigations on the fusion electrolysis with soluble and insoluble anodes to produce iron, chromium and manganese powders (34). Such methods have now become interesting for the production and purification of titanium and zirconium. Another study concerned the production of anhydrous chlorides of manganese (35) and zirconium by reaction of powdered carbon ferro manganese with iron dichloride and of powdered silico zirconium with the same reagent (36). Some time was devoted to reactions of powdered solid alloys with various chlorides to extract unwanted constituents (37). In this way copper could be eliminated from Monel metal turnings, iron from ferro nickel by reaction with fused nickel chloride in a carrier salt (38). Another investigation referred to the separation of tantalum and niobium in the oxides, by hydrogen reduction followed by chlorination (39), and by ammonia nitriding and chlorination (40). I gave advice in projects concerning the production of silico thermic magnesium and of electrolytic calcium as well as for the reduction of uranium oxide with calcium. Another study brought out a process for producing sodium from its chloride in mixture with silicon and lime in a vacuum (41), which later was used on an industrial scale for some time.

ZIRCONIUM

In the summer of 1944 I made a call to the U. S. Bureau of Mines in Washington, D. C., to inquire about the status of titanium research in the laboratories of this institution which had taken up titanium

reduction as an investigation project. On this occasion I suggested to Dr. R. S. Dean, then Assistant Director, that I would be in a position to produce zirconium in a similar way on the basis of my experience in Luxembourg and that I would achieve this result in about six months, given sufficient help and equipment. I was told that the Albany, Oregon station of the Bureau had a zirconium-reduction program and that I would be welcome as a consultant in this new station.

In December 1945 I took over the zirconium project in Albany and thanks to the inspiring direction of the manager of this station, Dr. B. A. Rogers, and with the expert help of Dr. A. W. Schlechten and Mr. L. E. Yerkes, the first strip of zirconium was rolled in August 1946. Zirconium was quite difficult to produce (**42, 43, 44**). The metal is about three times as sensitive to nitrogen and oxygen as is titanium. It is pyrophoric when powdery. It cannot be comminuted by pounding under water because it reacts explosively with moisture under shock. Therefore aqueous extraction of the magnesium chloride after the reduction was out of the question and vacuum distillation of the salts, which I had practiced in Luxembourg, became imperative. The first batches that came out of this procedure burned down until the trick of air conditioning was found. Melting represented first great difficulties. Arc melting of buttons was used but the first metal was very hard. I observed that melting with graphite electrodes, even on a graphite plate, did not introduce more that 0.2 per cent carbon in zirconium and that this did not harden the metal appreciably. After that I decided to melt directly in a graphite crucible (**45**). This was a great success and a split tube graphite resistor furnace with self tapping crucible was put into operation. Only after zirconium was being used in atomic research did it appear that even a little carbon was obnoxious because it lowered the corrosion resistance. A switch had to be made to melting in a water-cooled copper sleeve as was already practiced at that time by various titanium producers. One year after the first 4-lb. batches of zirconium had been made, the first zirconium pilot plant with 75 lb. per reactor got under way. I did not want to expose the shell of the large vacuum retort at elevated temperature to the pressure of the atmosphere and so decided on a radical departure from a common practice by putting an electric vacuum furnace over the hot end of the retort. This method came to general use and at present batches of two tons of titanium are separated that way from their salts. A year later the actual 150-lb. reactor, which I drew up, was put into operation in a larger pilot plant (**46**).

The main idea of my zirconium process, namely that the raw chloride cannot be mixed with the magnesium, is based on a simple observation. When zirconium chloride, which has been exposed to air, is heated in a glass tube, slightly acid water droplets condense first at the cool end. Anhydrous zirconium chloride distills off later, thus

showing that the water present at the start does not hydrolyze the chloride, except on the surface, which retains it like silica gel. Under such conditions large losses of chloride occur in a heated can containing superficially hydrolyzed zirconium chloride because of the slow heat penetration. There is a risk of having water vapor originating from

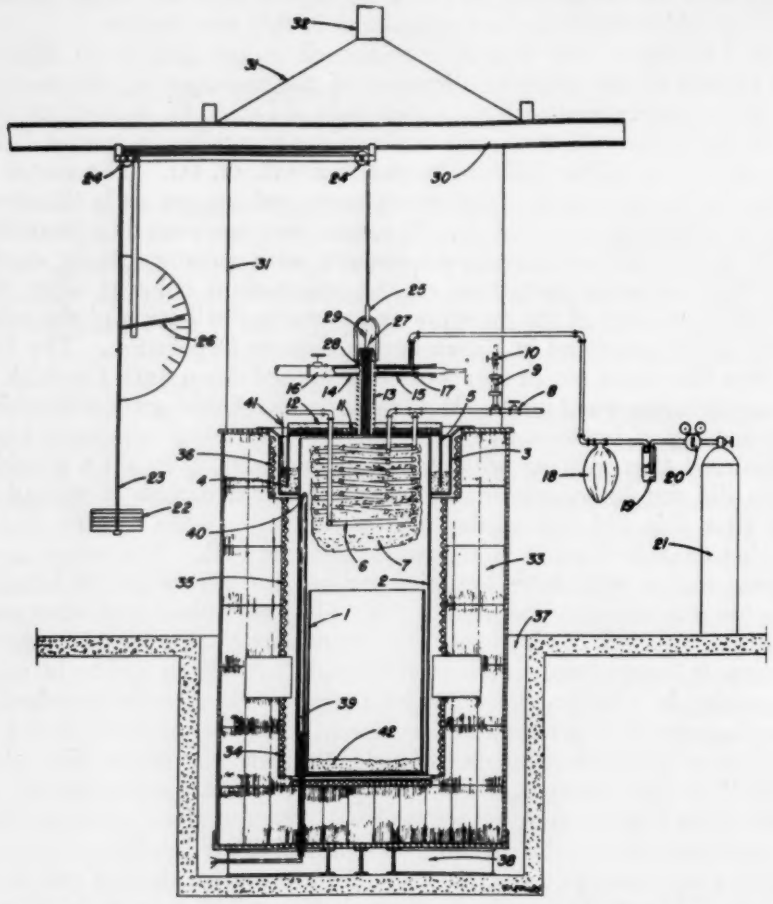


FIG. 12. Commercial equipment for purifying and reducing zirconium chloride. U. S. Bureau of Mines, Albany, Oregon.

the cool center of the mass, reacting with chloride subliming from the hot rim and forming oxide. A gas-phase purification of the chloride to eliminate any contained oxide is necessary. Furthermore, volatile iron trichloride is usually present in large enough quantities in the chloride to spoil the sponge. It has to be neutralized by some means. These two facts led to the construction of a reactor head, which was

provided with water-cooled coils on which subliming zirconium chloride was condensed in a hydrogen atmosphere. This gas reduces the volatile iron trichloride to the less volatile dichloride which remains in the residue with any ZrO_2. Since $ZrCl_4$, if completely sealed in a reactor, would develop a very high pressure if heated above 331° C., the subliming temperature, some safety escape for the gases had to be foreseen. That was easily achieved by using a fused lead seal with a floating condensor top. The solution adopted is shown schematically in Fig. 12.

I had no direct hand in the industrial development of the titanium process. This is entirely the merit of Mr. F. S. Wartman, but repercussions from innovations introduced in zirconium production were at once felt in the making of titanium. The vacuum distillation was immediately adopted on a large scale in titanium metallurgy and the melting in graphite is still recommended today for the production of castings and for the recovery of titanium scrap.

It has to be stressed that zirconium was made available in quantity just as it was needed in the production of atomic energy and for the construction of the first atomic submarine reactor. I had no part, however, in the elimination of hafnium from zirconium which is necessary in this application.

Some minor work was done at the Bureau on the production of lithium by vacuum reduction of the oxide in presence of CaO with aluminum (**47**), on the temperatures of reaction of carbon with various oxides *in vacuo* (**48**) and on the production of ductile chromium (**49**) by reduction of the chloride with magnesium and by purifying the metal with dry hydrogen passed over titanium hydride. I resigned from the Bureau in 1950.

SUMMARY

An appreciation of the advances in titanium production processes, based on historical data, is presented in Table I. As seen therefrom each investigator built up his work on that of his predecessors and the contribution of each seems rather small. The struggle for sodium-reduced metal is typical of this. Nilson and Petterson introduced the bomb method but they borrowed it from the current practice of reducing metal halides and oxides with alkali metals in a bolted gun barrel. Hunter just copied the Nilson-Petterson process and he was somewhat more successful. Good malleability of the titanium obtained was later achieved with this method by Lely and Hamburger. Pressureless sodium reduction was already used by Billy, before Freudenberg operated with this proposition on a large scale. It is up to the reader to judge my own contribution on the basis of the previous art given in Table I.

Shortly after the start of the war my U. S. titanium patent (**26**) was seized by the Alien Property Custodian. A five-year litigation

TABLE I.—*Historical Data About Marked Progress in Titanium-Reduction Methods.*

Year	Name	Ref.	Method	Quality of metal obtained: Ductile Hot	Ductile Cold	Brittle
1825	Berzelius	64	K_2TiF_6 reduced with K			X
1855	St. Claire Deville	65	$TiCl_4$ vapors passed over Na			X
1887	Nilson and Petterson	57	$TiCl_4$ reacted with Na in a bomb			X
1892	Seubert and Schmidt	66	Mg filing reacted with $TiCl_4$ under CO_2			X
1910	Hunter	58	$TiCl_4$ reacted with Na in a bomb like Nilson-Petterson	X		
1914	Lely and Hamburger	67	$TiCl_4$ reduced with Na in a bomb like Nilson-Petterson		X	
1919	Weiss	68	Dissociation of halides on a hot filament		X	
1921	Billy	69	$TiCl_4$ reduced $TiCl_4$ with Na under hydrogen; TiH_2 produced was degassed in vacuo			?
1925	Van Arkel and De Boer	70	TiI_4 dissociated on a hot filament		X	
1939	Freudenberg	71	Reduced $TiCl_4$ with Na in a chloride flux under hydrogen of atmospheric pressure			X
1940*	Kroll	26	Reduced $TiCl_4$ with fused Mg under argon of atmospheric pressure		X	

* Process first used in 1937.

followed. The high cost of this action in Court, the impairment of my health which it caused, the many years of inactivity it forced on me made me regret ever having started this ungratifying research work on titanium. The final court decisions will probably not allow me to recover my research expenditures and litigation costs from royalties, especially when considering taxes, possible infringement suits, substitute processes, continuously dropping titanium prices and the short life of my patent which expires in 1957. It is, however, a great satisfaction to have received from my colleagues here and abroad moral support and many honors, and to be able to admire the steady growths of the two big industries of titanium and zirconium to which a good fate allowed me to contribute a few ideas.

When in 1937 I set out alone with my rather limited means to solve the big problem of finding a cheap process of production for ductile titanium, I was a visionary, and I was considered as such by the outstanding metallurgists of the United States when I visited this country a year later. It was the enjoyment of discovery, the curiosity to know what is around the corner, which motivated my action and let me get over hardships and relative poverty, until final success was achieved. This example should refute all those who claim that today individual research—as was done by Pasteur or Fleming—is dead, and that it has been replaced by the collectivist planned work of teams or groups of salaried men, who operate the scientific machinery in private and

government laboratories in hopes of discovering something under the command of a director who, although usually removed from the observation of the experiments, is supposed to do the thinking. I do not agree with this claim. The free individual who accepts risks and years of sacrifice and who is driven by religious devotion to his task and by the spiritual joy which invention procures, in much the same way as is the artist by aesthetic achievements, can still today defy and outsmart the captive salaried research performed at extravagant cost in palatial laboratories where frequently nothing else is wanting but ideas. Money alone does not create or invent and the free individual still has great chances of success because his ideas originate in that same mind that commands the activity of his hands, while his thoughtful eyes watch the things that happen. This is the philosophy behind the invention of ductile titanium.

REFERENCES*

(1) *Zeit. anorg Chem.*, Vol. 102, pp. 1–33 (1918).
(2) Germ. Pat. 381,577, 1916 and 386,602, 1916; U. S. Pat. 1,707,059, 1929.
(3) Germ. Pat. 382,077, 1923; 381,049, 1920; 399,399, 1924; U. S. Pat. 1,359,813, 1921.
(4) Germ. Pat. 410,533, 1917; 615,566, 1930; 622,135, 1930; U. S. Pat. 1,428,041, 1922.
(5) *Metall. & Erz*, Vol. 19, p. 317 (1922).
(6) Germ. Pat. 423,245, 1921.
(7) *Metall & Erz*, Vol. 35, pp. 252–254, 282–286 (1938).
(8) Fr. Pat. 594,851, 1925.
(9) Germ. Pat. Appl. B 11,558 VI/40 b, publ. June 18, 1925.
(10) Germ. Pat. 443,641, 1925.
(11) *Metall Wirt.*, Vol. 13, pp. 725–731 (1934); *Metal Industry*, Vol. 47, pp. 3–6, 29–31, 103–105, 155–156 (1935).
(12) *Metall & Erz*, Vol. 23, pp. 555–557 (1926).
(13) *Ibid.*, pp. 225–230.
(14) *Ibid.*, pp. 613–616.
(15) *Ibid.*, pp. 682–685; Germ. Pat. 484,395, 1929.
(16) *Metall & Erz*, Vol. 23, pp. 590–594 (1926).
(17) *Zeit. anorg. Chem.*, Vol. 219, pp. 301–304 (1934).
(18) U. S. Bureau of Mines Information Circular 7327, 1945.
(19) *Metall Wirt.*, Vol. 17, pp. 463–466 (1938).
(20) *Zeit. Metallkunde*, Vol. 28, pp. 30–33 (1936).
(21) *Ibid.*, Vol. 23, pp. 33–34 (1931); Germ. Pat. 593,783, 1929; U. S. Pat. 1,991,162, 1935.
(22) Germ. Pat. 480,128, 1926; U. S. Pat. 1,740,857, 1930.
(23) *Zeit. anorg. Chem.*, Vol. 234, pp. 42–50 (1937).
(24) *Metall Wirt.*, Vol. 18, pp. 77–80 (1939); *Zeit. Metallkunde*, Vol. 29, pp. 189–192 (1937).
(25) *Trans. Electrochem. Soc.*, Vol. 78, pp. 35–47 (1940).
(26) U. S. Pat. 2,205,854, 1940.
(27) *Z.d.V.d.I*, Vol. 79, pp. 28–29 (1935).
(28) Germ. Pat. 604,337, 1934; Br. Pat. 352,964, 1929; *Metall Wirt.*, Vol. 9, pp. 1043–1045 (1930)
(29) U. S. Pat. 1,986,585, 1935; Germ. Pat. 537,716, 1930; Germ. Pat. 595,818, 1930.
(30) *Metall. Wirt.*, Vol. 11, pp. 31–32 (1932).
(31) *Met. & Alloys*, Vol. 20, p. 1604 (1944).
(32) *Zeit. Metallkunde*, Vol. 31, p. 20 (1939).

* References 1 through 50 are all by W. J. Kroll; the rest are general references.

(33) U. S. Pat. 2,246,886, 1942; 2,287,888, 1942; 2,310,094, 1943.
(34) *Trans. Electrochem. Soc.*, Vol. 87, pp. 551–567 (1945); U. S. Pat. 2,413,411, 1946.
(35) *Metal. Ind.*, Vol. 81, pp. 243–245 and 252, 269–271, 284–286, 307–311, 325–326, 341–343 and 365–366 (1952).
(36) U. S. Pat. 2,443,253, 1948.
(37) U. S. Pat. 2,452,665, 1948; Canad. Pat. 441,145, 1947.
(38) Canad. Pat. 441,144, 1947; 441,146, 1947; U. S. Pat. 2,396,794, 1946; U. S. Pat. 2,396,793, 1936; 2,396,792, 1946.
(39) U. S. Pat. 2,443,254, 1948.
(40) U. S. Pat. 2,427,360, 1948.
(41) U. S. Pat. 2,465,730, 1949.
(42) *Trans. Electrochem. Soc.*, Vol. 89, pp. 263–275 (1946).
(43) *Ibid.*, Vol. 92, pp. 90–113 (1947).
(44) *Ibid.*, Vol. 94, pp. 1–20 (1948).
(45) *Trans. AIME*, Vol. 175, pp. 766–773 (1948).
(46) *Ibid.*, Vol. 188, pp. 1445–1453 (1950).
(47) AIME Tech. Publ. No. 2179, Met. Tech. June 1947, 9 p.
(48) *Trans. Electrochem. Soc.*, Vol. 93, pp. 247–258 (1948).
(49) *Ibid.*, Vol. 97, pp. 258–264 (1950).
(50) Germ. Pat. 487,431, 1926.
(51) C. L. MANTELL, "Industrial Electrochemistry," New York, McGraw-Hill Book Co., 1940, pp. 448–450.
(52) H. CARON, *Compte Rendus*, Vol. 48, p. 440 (1859).
(53) T. E. LEONTIS AND J. P. MURPHY, *Trans. AIME*, Vol. 166, pp. 295–327 (1946).
(54) J. O. BETTERTON AND Y. E. LEBEDEFF, *Ibid.*, Vol. 121, pp. 205–225 (1936).
(55) J. O. BETTERTON, Germ. Pat. 648,839 and 650,332, 1937.
(56) Heraeus Vacuumschmelze, Anniversary 1923–1933. Albertis, Editor, Hanau/Main.
(57) L. F. NILSON AND O. PETTERSON, *Zeit. phys. Chem.*, Vol. 1, pp. 27–28 (1887).
(58) M. A. HUNTER, *J. Amer. Chem. Soc.*, Vol. 32, pp. 330–336 (1910).
(59) B. R. F. KJELLGREN, *Trans. Electrochem. Soc.*, Vol. 93, pp. 122–128 (1948).
(60) R. S. DEAN, J. R. LONG, F. S. WARTMAN AND E. I. ANDERSON, *Trans. AIME*, Vol. 166, pp. 369–381 (1946). F. S. WARTMAN, H. C. FULLER, J. P. WALKER, M. A. COOK AND E. L. ANDERSON, Bureau of Mines R.I. 4519, 1949.
(61) W. VON BOLTON, Refer to: W. ESPE AND M. KNOLL, "Werkstoffkunde der Hochvakuum Technik," Berlin, Springer, 1939, pp. 36–37.
(62) G. GRUBE, K. BAYER AND H. BUMM, *Zeit. Elektroch.*, Vol. 42, pp. 805–815 (1936).
(63) R. S. DEAN, "Electrolytic Manganese," New York, Ronald Press, 1952.
(64) J. J. BERZELIUS, *Pogg Ann. der Chemie*, Vol. 4, pp. 1–22 (1825).
(65) H. ST. CLAIRE DEVILLE, *Compte Rendus*, Vol. 40, pp. 1034–1036 (1855).
(66) K. SEUBERT AND A. SCHMIDT, *Liebigs Ann.*, Vol. 267, pp. 234–235 (1892).
(67) D. LELY AND L. HAMBURGER, *Zeit. anorg. Chem.*, Vol. 87, pp. 209–228 (1914).
(68) L. WEISS, Germ. Pat. 314,791, 1919.
(69) M. BILLY, *Ann. Chim. Pharm.*, Ser. 9, Vol. 16, pp. 5–54 (1921).
(70) A. E. VAN ARKEL AND J. H. DE BOER, *Zeit. anorg. Chem.*, Vol. 148, p. 345 (1925).
(71) H. FREUDENBERG, U. S. Pat. 2,148,345, 1939.
(72) L. WEISS AND E. NEUMANN, *Zeit. anorg. Chem.*, Vol. 65, pp. 262–264 & 328–333 (1910).
(73) W. T. ISBELL, *Trans. AIME*, Vol. 182, pp. 186–190 (1949).

MATRICES IN ANALOGUE MATHEMATICAL MACHINES

BY

PIERRE M. HONNELL[1] AND ROBERT E. HORN[1]

ABSTRACT

This paper initiates a rational account of the methods of solution of linear differential equations on electronic analogue machines from the matrix point of view. The 1-to-1 reciprocal correspondence between the mathematical problem matrices and the electronic synthesis networks is established. The systematic matrix viewpoint bearing on both the mathematical and physical systems yields theoretical and practical advantages, most of which remain to be exploited.

I. INTRODUCTION

Although germane to other types, the sequel is limited to the modern electronic analogue mathematical machine devised for the solution of ordinary linear differential and algebraic equations with constant coefficients. The purpose of this paper is to initiate, within these limits, a rational account of the basic theory of these machines devoid of the jargon which has arisen in their description and rules for their operation. This is achieved by a systematic application of the matrix point of view both to the mathematical and to the physical systems under consideration.

In a strict sense, an electronic analogue machine does not "solve" a mathematical problem; rather, it exhibits the implicit solution. For example, the "solution" to a set of n-th order differential equations is not obtained by a process of repeated quadratures by the use of so-called electronic "integrators" and "adders." Actually, the process comprises the reduction of the given system of differential equations to an equivalent first order system, and then the synthesis of the latter by means of an electronic network which exhibits the electric variables corresponding to the desired "solution."

Moreover, it is generally desirable further to transform the new first order system (equivalent to the original set of n-th order differential equations) so as to suit the characteristics of any particular electronic analogue machine. This is done for practical considerations, such as scaling in amplitude range, or time; and also for stability. The class of transformations is in fact at present dictated largely by considerations of the electronic network stability over and above the stability of the mathematical problem as such. This results from the fact that the differential equations governing the equilibrium of the electronic analog machine are not identical to the desired problem. Thus, the electronic network is often unstable although the desired mathematical problem is itself stable.

[1] Schwentler Computing Laboratory, Washington University, St. Louis, Missouri.

The important objective, therefore, is to arrange matters such that over-all stability is determined solely by the mathematical problem, and that the machine solution and the problem solution do not deviate by more than any preassigned quantity. This objective is not yet satisfactorily solved, although a more general matrix analysis does provide a systematic approach to the converse problem.

In any event, the special matrix formulation considered herein yields the great practical advantages of wide flexibility in analogue representations of a given mathematical problem, and of a rational choice of those scale factors and classes of network configurations known from experience to lead to practical results for any given machine.

The reason analogue machines are more readily adaptable to the representation of systems of algebraic and differential equations than are other electric network devices is also placed in a clearer light by the matrix viewpoint. The key to the matter is that a triode electron tube is a primitive trinode network element of higher order—and therefore of greater potentialities—than dinode network elements: that is, primitive ideal linear positive resistors, capacitors and inductors. As trinode network elements, the basic admittance matrices of electron tubes singly (or in suitable combinations) are of the second order. Furthermore, they may be constrained to operating ranges such that their admittance matrices (to a first approximation) have zeros in two or three of their entries.

The net result is that an arbitrary non-symmetrical admittance matrix with a random arrangement of null entries can be synthesized with suitable combinations of these electron-tubes and dinode admittances. This is *not* possible with networks consisting solely of linear dinode admittances, no matter how complex their interconnections. But as it turns out, this very general type of matrix (which can be synthesized with the electronic networks) arises naturally in the matrix formulation of first-order equivalents to systems of linear differential equations. This is the *raison d'être* of the electronic analogue machines.

The following sections bring into focus the 1-to-1 correspondence which can be established between mathematical problem matrices and the electronic analogue machine admittance matrices. To this end, accordingly, Section II recites briefly examples of the reduction of n-th order linear differential equation systems to equivalent first-order systems, and some of the various transformations which can be effected upon them. Section III establishes the new matrical formulation of the electronic analogue system equations. This is followed in Section IV by several examples which emphasize the application of the correspondence between the mathematical problem matrices and the associated electronic network admittance matrices synthesized for the display of the problem solutions.

II. RELEVANT MATHEMATICAL ASPECTS

1. *Algebraic Systems*

A set of linear algebraic equations may be written concisely as

$$Ax = b, \tag{1}$$

where A is the square matrix of the coefficients with real elements, x is a column matrix of unknowns, and b is a column matrix of real numbers. Since the transformation of linear differential equations for processing on electronic analogue machines includes submatrices of the form (1), for the sake of brevity in this paper, no further reference will be made to algebraic systems as such.

2. *Linear Ordinary Differential Equations*

The class of equations considered may be expressed compactly in matrix notation as

$$(A_1D^0 + A_2D + \cdots + A_nD^{n-1} + A_{n+1}D^n)x = f(t), \tag{2}$$

where

$D^n = Id^n$ and $D^0 = I$
d^n is written for d^n/dt^n
I is the m-th order identity matrix
A_i is an m-th order matrix of real constants with at least one non-zero element
x is a column matrix of m dependent real functions of t, to be determined
$f(t)$ is a column matrix of m-th order with prescribed real functions of t as elements
t is the independent real variable.

Equation 2, a system of ordinary differential equations with constant coefficients in m variables, with at least one of the equations of the n-th order, must be processed for convenience in machine applications. This is done as indicated in the following paragraphs.

The differential equation system (2) may be transformed (1, 2)[2] into an indefinitely large variety of equivalent first-order systems. Particularly useful among these for electronic analogue machine applications, as will become evident from Section III, are the following:

$$\begin{bmatrix} D & -I & 0 & \cdots & 0 & 0 \\ 0 & D & -I & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & D & -I \\ A_1 & A_2 & A_3 & \cdots & A_n & A_{n+1} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ x_n \\ x_{n+1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ 0 \\ f(t) \end{bmatrix} \tag{3}$$

[2] The boldface numbers in parentheses refer to the references appended to this paper.

in which

$$x_i = D^{i-1}x, \qquad i = 1, \ldots, n, n+1$$

and

$$\begin{bmatrix} D & -I & 0 & 0 & \cdots & 0 & 0 \\ 0 & D & -I & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots & D & -I \\ A_1 & A_2 & A_3 & A_4 & \cdots & A_{n-1} & (A_n + A_{n+1}D) \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ 0 \\ f(t) \end{bmatrix}. \tag{4}$$

in which

$$x_i = D^{i-1}x, \qquad i = 1, \ldots, n.$$

Premultiplication of (3) and (4) by the transformation

$$\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & I \\ -I & 0 & 0 & \cdots & 0 & 0 \\ 0 & -I & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & -I & 0 \end{bmatrix} \tag{5}$$

yields in the case of (3) the equivalent system

$$\begin{bmatrix} A_1 & A_2 & A_3 & \cdots & A_n & A_{n+1} \\ -D & I & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & I & 0 \\ 0 & 0 & 0 & \cdots & -D & I \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ x_n \\ x_{n+1} \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \\ \cdot \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

and in the case of (4) the equivalent system

$$\begin{bmatrix} A_1 & A_2 & A_3 & A_4 & \cdots & A_{n-1} & (A_n + A_{n+1}D) \\ -D & I & 0 & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots & I & 0 \\ 0 & 0 & 0 & 0 & \cdots & -D & I \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \\ \cdot \\ 0 \\ 0 \end{bmatrix}. \tag{7}$$

The principal characteristics of these equations which are of interest in machine operations may be summarized as follows:

(*a*) All m dependent variables and their derivatives including the n-th are present in Eqs. 3 and 6

(*b*) All m dependent variables and their derivatives *exclusive of the* n-th are present in Eqs. 4 and 7

(*c*) Equations 3 and 4 exhibit the differential coefficient submatrices $D^n = Id^n$ on their principal diagonals

(*d*) Equations 6 and 7 exhibit submatrices of constant elements on their principal diagonals.

The electronic network attributes corresponding to these mathematical properties will become clear from the following sections.

Of course, Eqs. 6 and 7 may be written directly from 2 without recourse to the transformation 5; the latter, however, exemplifies the convenience of matrix transformation in arriving at desired forms for equivalent systems of differential equations.

Another very useful transformation is that of changing dependent variables. Even the apparently trivial matter of change of scale indicated by

$$x = Cy \tag{8}$$

has important consequences in an actual electronic machine, as will become evident.

This brief account of specific examples of equivalents to the system of equations (1) has been chosen to illustrate certain facets of the historical development of electronic analogue machines. Clearly, the rich variety of such equivalent systems provides many properties useful either from the mathematical or the physical representation of differential equations. The discovery of the best transformations for any particular problem is an ever-present challenge, and is dictated as much by the inherent characteristics of the analogue machine as by the mathematical problem.

III. ELECTRONIC NETWORK ASPECTS

The basic problem in an analogue machine is to establish equilibrium equations of the physical system which correspond substantially to the differential equations to be studied. This is of course network synthesis. To be practicable, the synthesis procedure must be simple and direct. Now it is well known that the dinode synthesis of networks corresponding to differential equations—when this is possible—is inevitably accompanied by an annoying interlocking phenomenon. That is, the non-diagonal entries of the matrix of the mathematical equations determine not only the dinode network elements corresponding thereto, but also affect the magnitude of the network elements corresponding to the principal diagonal terms. This precludes a direct correspondence between single elements of the mathematical and the physical systems.

In electronic analogue machines, this interlocking effect is eliminated by virtue of the properties of the trinode electronic amplifiers. That is, each entry in the matrix of the equations corresponds directly to only one single network element, and reciprocally. Furthermore, non-symmetrical matrices, and matrices with null entries on the principal diagonals, may also be synthesized.

Ideally, the electronic amplifiers for analogue machine applications would enjoy the following properties over the range of interest of the variables:

(*a*) Unilateral voltage amplification which can be specified by large numbers $+\mu_k$ and $-\mu_k$
(*b*) Zero input admittance
(*c*) Indefinitely large output admittance.

The dinode admittances of the machine are similarly postulated as ideal linear, positive, conductances and capacitances. Other dinode admittances are employed in practice, but are excluded herein.

Electronic analogue machines use the above-mentioned network elements in the configuration indicated in Fig. 1 as the fundamental synthesis network. This electronic network has an admittance matrix —on the common-node coordinate basis indicated in Fig. 1—which

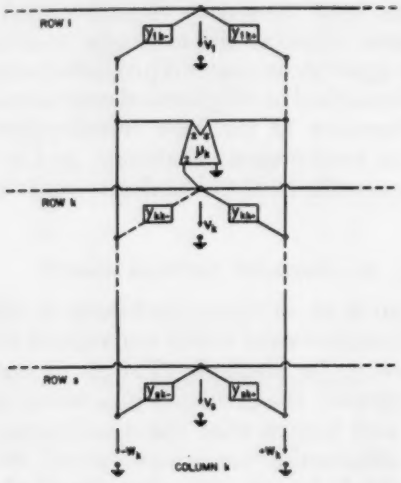


FIG. 1. The fundamental electronic synthesis network.

reads ideally as

$$\begin{bmatrix} 0 & \cdots & 0 & \mu_k(y_{1k+} - y_{1k-}) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \mu_k(y_{2k+} - y_{2k-}) & 0 & \cdots & 0 \\ \cdot & \cdots & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & \cdots & 0 & \mu_k(y_{kk+} - y_{kk-}) & 0 & \cdots & 0 \\ \cdot & \cdots & \cdot & \cdot & \cdot & \cdots & 0 \\ 0 & \cdots & 0 & \mu_k(y_{rk+} - y_{rk-}) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \mu_k(y_{sk+} - y_{sk-}) & 0 & \cdots & 0 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ v_k \\ \cdot \\ v_r \\ v_s \end{bmatrix} = \begin{bmatrix} j_1 \\ j_2 \\ \cdot \\ j_k \\ \cdot \\ j_r \\ j_s \end{bmatrix}, \tag{9}$$

in which the j_k are prescribed current sources, and the v_k are the electronic network dependent variables, called for convenience the "(mathematical) problem variables." The dinode admittances y_{rk} are constants, or those functions of $d = d/dt$ obtainable with interconnections of the primitive dinode elements previously mentioned.

Although the dinode coupling elements y_{kk-} yield completeness and

symmetry to the mathematical expressions, experience indicates that these coefficients often cannot be included in physical machines due to instabilities which arise. Hence the connections for these admittances are shown dotted in Fig. 1. Limitations of the current electronic art also preclude the use of intrinsic prescribed current sources, the j_k; rather, their equivalents constructed from intrinsic voltage sources are employed instead.

The problem variables v_k exhibiting the desired implicit solutions are represented by an aggregation of these synthesis networks. As is clear from (9), the presence of the μ_k, which are ideally large real numbers, means that the voltage magnitudes of the variables v_k are intrinsically small in a physical sense and therefore difficult to record oscillographically. The amplifying elements in the synthesis networks themselves, however, also provide a means for displaying the problem variables at a more useful magnitude level.

The idea is most compactly indicated by the change of variable

$$v = Cw, \tag{10}$$

where

$$C = \begin{bmatrix} \frac{1}{\mu_1} & 0 & \cdot & 0 & \cdot & 0 & 0 \\ 0 & \frac{1}{\mu_2} & \cdot & 0 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \frac{1}{\mu_k} & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 0 & \cdot & \frac{1}{\mu_r} & 0 \\ 0 & 0 & \cdot & 0 & \cdot & 0 & \frac{1}{\mu_s} \end{bmatrix} \tag{11}$$

which relates the amplifier input variables v_k (the problem variables) to the amplifier output variables w_k, called for convenience the "display variables." It should incidentally be noted that the μ_k need not all be equal, although all are large.

Substitution of the transformation (10) into Eq. 9 results in the system

$$\begin{bmatrix} 0 & \cdots & 0 & (y_{1k+} - y_{1k-}) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & (y_{2k+} - y_{2k-}) & 0 & \cdots & 0 \\ \cdot & \cdots & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & \cdots & 0 & (y_{kk+} - y_{kk-}) & 0 & \cdots & 0 \\ \cdot & \cdots & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & \cdots & 0 & (y_{rk+} - y_{rk-}) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & (y_{sk+} - y_{sk-}) & 0 & \cdots & 0 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ w_k \\ \cdot \\ w_r \\ w_s \end{bmatrix} = \begin{bmatrix} j_1 \\ j_2 \\ \cdot \\ j_k \\ \cdot \\ j_r \\ j_s \end{bmatrix}. \tag{12}$$

The resulting matrix equation of the physical system (12) is now of the desired form, and relates the display variable w_k to the prescribed

time function j_k by an admittance matrix in which the voltage amplification μ_k of the respective electronic amplifier no longer appears.

The electronic synthesis network matrix (12) has the same form as *one column* of a mathematical matrix, such as (3), (4), (6) or (7), null entries in these being represented in the synthesis network by zero-valued y_{rk}. An $(\bar{s},s)$ order mathematical matrix therefore requires **s** synthesis networks for its representation, one for *each* column. The sum of the corresponding **s** synthesis matrices (12)—in view of the zero entries in the latter—then forms an ensemble identical in form to the complete mathematical problem matrix.

This establishes the 1-to-1 reciprocal correspondence between the electronic machine equilibrium equations and the mathematical problem. Furthermore, the *problem matrix itself* can be used directly in synthesizing the electronic network. That is, the problem matrix—interpreted in terms of the variables in (12)—contains within its structure the topological schema of the interconnections of the machine. This completely eliminates the necessity for any form of connection diagram. Since connection diagrams do serve a useful purpose and are commonly associated with electronic analogue machines, an example of a synthesis network schema follows together with an ideographic representation compatible with the matrix viewpoint.

The above considerations may be illustrated in general form by an example of a single n-th order linear differential equation with constant coefficients, of the form (1). The latter may be written as indicated by (4), in the following way:

$$\begin{bmatrix} d & -1 & 0 & 0 & \cdot & 0 & 0 \\ 0 & d & -1 & 0 & \cdot & 0 & 0 \\ 0 & 0 & d & -1 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdot & d & -1 \\ a_1 & a_2 & a_3 & a_4 & \cdot & a_{n-1} & (a_n + a_{n+1}d) \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ 0 \\ f(t) \end{bmatrix} \tag{13}$$

in which the coefficients a_k are considered positive for concreteness.

This mathematical problem matrix, (13), may be couched in terms of electric parameters and variables in which case the corresponding admittance matrix reads

$$\begin{bmatrix} y_{11+} & -y_{12-} & 0 & \cdots & 0 & 0 \\ 0 & y_{22+} & -y_{23-} & \cdots & 0 & 0 \\ 0 & 0 & y_{33+} & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & y_{(n-1,\,n-1)+} & -y_{(n-1,\,n)-} \\ y_{n1+} & y_{n2+} & y_{n3+} & \cdots & y_{(n,\,n-1)+} & y_{(n,\,n)+} \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \cdot \\ w_{n-1} \\ w_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ 0 \\ j_n \end{bmatrix} . \tag{14}$$

Each entry of the matrix (14) is defined as to numerical magnitude by the corresponding entry in the mathematical problem matrix (13).

The synthesis network schema associated with Eq. 14 is shown in

Fig. 2, and is more compactly depicted by the ideograph in Fig. 3. The trinode amplifiers in the latter are indicated by the squares on the principal diagonal, each node voltage or problem variable v_k being connected to the respective amplifier input on each row, and the corresponding display variables $+w_k$ and $-w_k$ appearing in the respective columns.

Clearly, the dinode admittance magnitudes shown on these figures are unrealistic practically. This is of small moment; for, multiplying every row of the matrix (14) by a constant factor, such as 10^{-6}, results

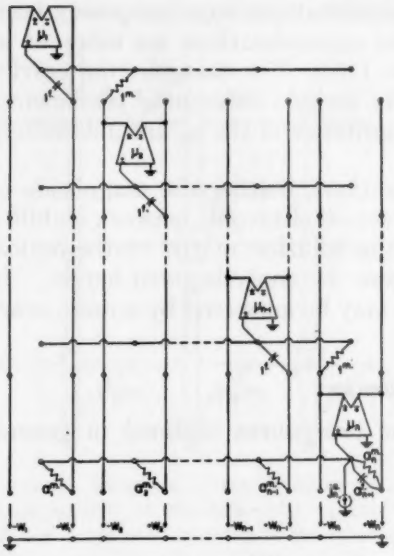


FIG. 2. Schema of the electronic synthesis network of the n-th order linear differential equation.

Legend: f.—farad
m.—mho
a.—ampere

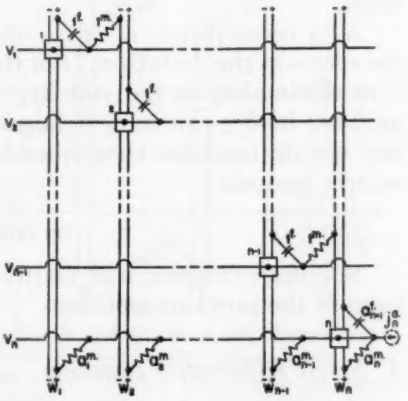


FIG. 3. Ideograph of the electronic synthesis network of the n-th order linear differential equation.

Legend: f.—farad
m.—mho
a.—ampere

in coefficients and variables within the commonly realizable ranges, namely: seconds, micromhos, microfarads, volts, and microamperes. Although the simplest procedure is to multiply all the rows of (14) by the same constant factor, this is by no means necessary, for each row may have its own multiplying factor. In practical situations, this may be very advantageous. Similar considerations apply to change in scale of the dependent variables, which results when columns are multiplied by a constant. The equivalence of the indicated solutions of the modified and of the original problem is maintained by these elementary transformations.

Numerical or analogue solutions of differential equations exist only for specified initial or boundary conditions. In electronic analogue machines, the boundary conditions are specified initially. These are entered by introducing the corresponding problem node variables initially at the specified voltage corresponding to their prescribed initial magnitudes. In effect, initial conditions introduced in this manner represent a switch-off transient solution. To this is added the response resulting from the prescribed functions of the independent variable, the j_k, which themselves are discontinuous.

To maintain this analysis within the bounds of its objectives, and for the sake of simplicity, certain approximations have purposely been submerged from view. The principal approximations are inherent in Eq. 9 and the transformation matrix (10). For example, the entries in (11) are not constants, but actually include differential coefficients. Similarly, (9) is exact only if the magnitudes of the μ_k are indefinitely large.

As a consequence of these idealizations, neither the magnitude of the errors in the "solution," nor the over-all electronic network stability —in distinction to the stability of the solution of the mathematical problem itself—can be determined from the analysis given herein. In any specific instance, these questions may be answered by a more exact matrix analysis.

IV. TYPE PROBLEMS

Specific examples will clarify the procedures outlined in general form in the previous sections.

1. Single Differential Equation

As representative of the n-th order linear differential equation with constant coefficients, consider the single equation

$$(d^2 + d + 1)x = 0. \tag{15}$$

According to the procedure outlined in Section II, this equation may be written immediately in the equivalent first-order system form (3),

$$\begin{pmatrix} d & -1 & 0 \\ 0 & d & -1 \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{16}$$

in which the display voltage variables w_1, w_2, and w_3 in (16) correspond to x, dx, and d^2x in the problem Eq. (15).

As discussed in Section III, the synthesis of Eq. 16 is immediate, and yields the electronic network ideograph Fig. 4. The 1-to-1 correspondence between the matrix (16) and this ideograph is clearly evident.

The transformation matrix (5) applied to problem (16) reads

$$\begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \tag{17}$$

and results in a second equivalent to (15), namely

$$\begin{pmatrix} 1 & 1 & 1 \\ -d & 1 & 0 \\ 0 & -d & 1 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \tag{18}$$

The corresponding ideograph is shown in Fig. 5.

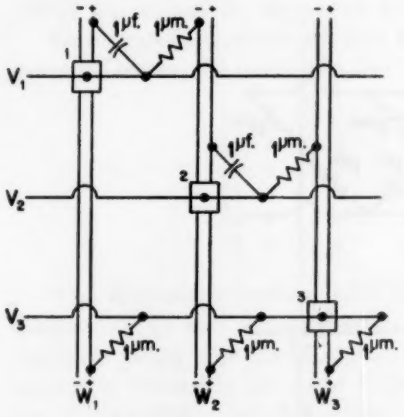


FIG. 4. Ideograph of the electronic synthesis network of the differential equation $(d^3 + d + 1)x = 0$, according to the scheme (16).

Legend: μf.—microfarad
μm.—micromho

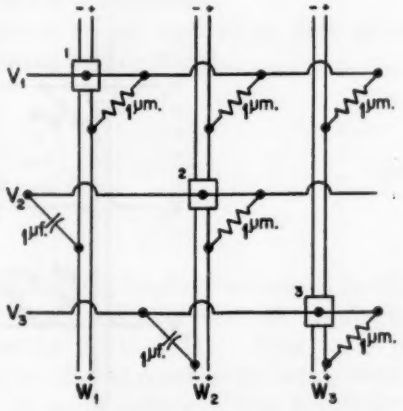


FIG. 5. Ideograph of the electronic synthesis network of the differential equation $(d^3 + d + 1)x = 0$ according to the scheme (18).

Legend: μf.—microfarad
μm.—micromho

Equations 16 and 18 are of the forms (3) and (4) respectively, and entertain the properties mentioned in Section II. In current terminology, these are the so-called "integrator" and "differentiator" techniques of obtaining the "solution" of problem (15). This conventional nomenclature is clearly misleading, for the only significant distinction between Figs. 4 and 5 is that the former has differential coefficients and the latter constant terms on the principal diagonals of the respective matrices and networks. The reason for the usual preference of the form (16) over (18) with existing machines actually resides in the inherent approximations previously mentioned; and an analysis including these residuals will show that form (16) leads to a stable electronic synthesis network, whereas (18) is often unstable.

A third first order system equivalent to the differential equation

(15) is obtained from (16) by premultiplying the latter by

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{19}$$

This yields the equivalent system

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & d & -1 \\ d & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{20}$$

with the corresponding electronic network configuration shown in Fig. 6. The matrix (20) has been synthesized on an experimental

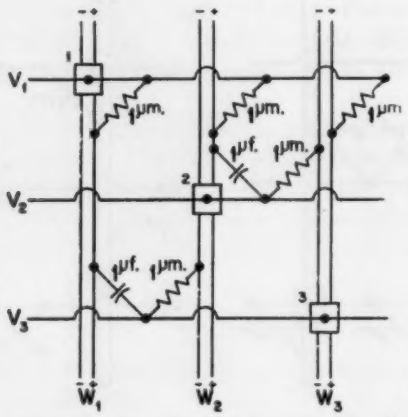


FIG. 6. Ideograph of the electronic synthesis network of the differential equation $(d^2 + d + 1)x = 0$ according to the scheme (20).

Legend: μf.—microfarad
μm.—micromho

electronic analogue machine and its stable operation verified. The exact nature of the solution determined experimentally was in agreement with the results of a more complete analysis including the approximation terms not indicated in (20).

Figure 6 is illustrative of one of the variety of electronic network configurations which exhibit the solution to Eq. 15 but which would not readily be found by the existing conventional approach which yields Figs. 3 and 4. This is not surprising, for the conventional approach has no rational means of arriving at the equivalent mathematical system (20).

2. *Systems of Differential Equations*

The processing of a system of differential equations for an electronic analogue machine according to the methods described provides an

interesting view of the power of the matrix approach to this more involved problem. An example is the system of differential equations

$$\begin{aligned} d^2x + d^2y - d^2z + dy + x + y &= 0 \\ dy + z &= 0 \\ 2\,dx + dz + z &= 0 \end{aligned} \tag{21}$$

which may be written in the notation of Eq. 2 as

$$\left(\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} D + \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} D^2 \right) \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{22}$$

where w_1, w_2 and w_3 represent x, y, and z, respectively.

One transformation of this equation to an equivalent first order system (see Eq. 4) results in the following arrangement:

$$\begin{bmatrix} d & 0 & 0 & -1 & 0 & 0 \\ 0 & d & 0 & 0 & -1 & 0 \\ 0 & 0 & d & 0 & 0 & -1 \\ 1 & 1 & 0 & d & (d+1) & -d \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{23}$$

The first-order system (23) exhibiting the implicit solutions to the problem (21) can be synthesized directly on an electronic analogue machine using the methods outlined in Section III. This form of matrix is unusable on some types of available commercial equipment, due to instability of the machine. In such instances, the instability can be traced to the presence of differential coefficients, the *d:s*, on other than the principal diagonal elements of the matrix of the equations.

A stable form of equation may be obtained by the following pre-multiplier acting on Eq. 23,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -d & d \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{24}$$

This yields the desired system equivalent to the differential Eqs. 21 with the *d:s* appearing only on the principal diagonal of the matrix, which follows:

$$\begin{bmatrix} d & 0 & 0 & -1 & 0 & 0 \\ 0 & d & 0 & 0 & -1 & 0 \\ 0 & 0 & d & 0 & 0 & -1 \\ 1 & 1 & 0 & 3d & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{25}$$

The corresponding electronic synthesis network appears in Fig. 7.

By formulating the problem (21) according to the scheme (4), the x_{n+1} dependent variables of (22) do not appear in (23) nor (25). If all the variables and their derivatives given in (21) had been desired, the scheme (3) could be followed. In this event, the submatrix A_{n+1} would display two zero entries on its principal diagonal. The synthesis of the resulting matrix is again perfectly straight-forward; but this too commonly yields an unstable electronic network in many available machines. Depending upon the particular machine, a suitable transformed system could probably be found which would yield a stable electronic synthesis network and exhibit all the desired dependent variables.

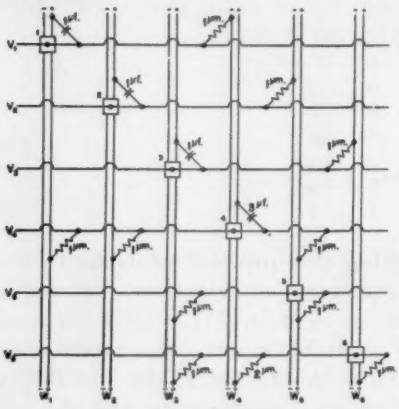


FIG. 7. Ideograph of the electronic synthesis network of the differential equation system (21) according to the scheme (25).

Legend: μf.—microfarad
μm.—micromho

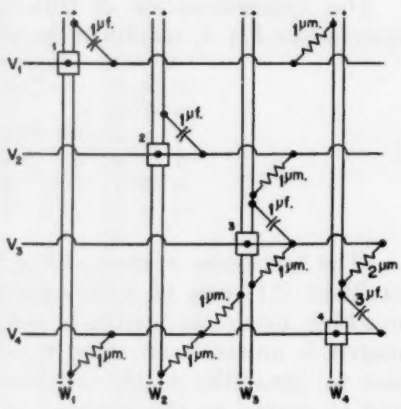


FIG. 8. Ideograph of the reduced system (26) equivalent to the differential equation system (21).

Legend: μf.—microfarad
μm.—micromho

Finally, if the dependent variables x, y and z only of (21) are desired, and not their derivatives, it is evident from (25) that an additional simple transformation eliminating the last two equations would give the fourth order matrix equation

$$\begin{pmatrix} d & 0 & 0 & -1 \\ 0 & d & 1 & 0 \\ 0 & 0 & d+1 & 2 \\ 1 & 1 & -1 & 3d \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} . \tag{26}$$

The ideograph of the electronic synthesis network corresponding to (26) is shown in Fig. 8. It is true that this matrix yields $dx \approx w_4$ in addition to x, y and z, but this was chosen because a fourth-order

matrix is the lowest order system equivalent to (21) which preserves first-degree differential coefficients, or *d:s*, in the matrix elements.

Finally, it should be noted that the examples of matrix manipulations indicated above do not exhaust all possibilities of this type of elementary transformation. For example, change of scale of the independent variable, and the adjoining of additional electronic elements to form higher ordered synthesis networks, are also very useful transformations yielding equivalent systems.

Future experience will undoubtedly yield new types or combinations of transformations which will be useful from the theoretical and practical points of view.

V. CONCLUSIONS

This paper gives only a brief indication of the power of the matrix viewpoint as applied to electronic analogue machines. One real merit of the matrix method has been amply demonstrated under the stress of application to the solution of practical problems, in which the systematic approach resulted in considerable reductions in set-up time and computing time.

From the theoretical side, the matrix approach shows promise of yielding new and useful results both in the transformation of problems specifically for machine applications, and in the synthesis of new types of electronic networks.

Clearly, the results to date yield but a preliminary glimpse of the potentialities of the new methods. An assessment of the full potency of the matrix method applied to electronic analogue machines cannot be forecast without further research.

REFERENCES

(1) E. L. INCE, "Ordinary Differential Equations," Dover Publications, Incorporated, New York.

(2) R. A. FRAZER, W. J. DUNCAN, AND A. R. COLLAR, "Elementary Matrices," Cambridge University Press, London, 1938.

Automatic Spectrophotometer Program Control.—Applied Physics Corporation, Pasadena, Calif., has announced the development of an automatic spectrophotometer program control which permits repetition of a spectral scan at pre-determined intervals over a 24-hr. period, without attention from an operator.

According to the announcement, the program control can be used with Cary Spectrophotometers to follow the course of reactions, in reaction rate studies, in continuous analysis of flowing samples, in analysis of chromatographic effluents, etc.

Some laboratories are said to gain added usage from their instruments by using them during the day on routine work, then setting them up with the program control for automatic operation during the night.

In using the program control, the wavelength limits of the region to be scanned are set into the panel controls on the spectrophotometer, and time intervals are set into the control unit. Other conditions of the analysis are set up on the spectrophotometer in the usual manner.

The spectrophotometer then automatically scans between the limits set, returning after each scan to the starting conditions in readiness for the next scan. Spectra appear in sequence on the recorder chart paper.

Spring-mounted Jewels.—Spring-mounted jewels affording unique protection against shock and pivot wear are now a feature on all General Electric portable indicating instruments. The company's Instrument Department at Lynn, Mass., said the use of spring mountings, with pivot retaining collars, prevents the blunting or "mushrooming" of pivots due to jarring of the instruments, which are used for various types of voltage and current measurements. This affords protection against the need for extensive and expensive repair often occasioned by the blunting of pivots.

Two types of G-E portable testing instruments—the DP11 and DP12, have had spring-mounted jewels since their introduction in 1953. Now this exclusive feature is incorporated in G-E's most popular portable instrument lines—types P-3, AP9 and DP9.

Plug-in Tubular Capacitors for Printed Circuits.—Cornell-Dubilier Corp. (South Plainfield, N. J.) announces the development of its new "Type BC" phenolic-cased plug-in paper tubular capacitors especially designed for use with printed circuits. These new capacitors are encased in molded phenolic shells with two parallel lead wire terminals. These terminals are brought out from the end of the capacitor through a thermo setting plastic end fill compound, and are spaced a fixed distance so that they may be plugged directly into printed circuits and dip soldered.

Distortion and stresses inherent in pressure-molded assemblies are entirely eliminated by inserting the capacitor section in a pre-molded, mineral-filled phenolic shell. The capacitor element, which is wound from pure aluminum foil and high quality tissue, is sealed within the case by C-D Polykane which bonds to the container wall and wire leads, barring moisture creepage, while at the same time holding the lead wires rigidly in place. Polykane will not soften, melt, or flow; even under the extremes of soldering temperatures. Low resistance electrical connections are assured by soldering the lead wires directly to the extended foils of the capacitor element.

COMPLEX ZEROS OF THE ERROR FUNCTION

BY

HERBERT E. SALZER[1]

The error function $f(z) = \int_0^z e^{-u^2}\,du$ has only complex zeros in addition to the obvious real zero $z = 0$. These complex zeros z_n, $n = 1,2, \ldots,$ which are infinite in number, are symmetrically situated in the four quadrants of the z-plane. Thus it suffices to consider only those within the first quadrant, all of which lie above the 45° ray, approaching it asymptotically as $n \to \infty$. The only previous attempt at tabulation was by T. Laible (1)[2] who gave the first five complex zeros to $3D$. They were obtained graphically and his 4th and 5th zeros are inaccurate even in the second decimal place. Because of the fundamental importance of the error function in both pure and applied mathematics, it was believed that there might be interest in a more extensive tabulation of its zeros.

TABLE I.

n	z_n
1	1.45061 61632 44 + 1.88094 30001 53i
2	2.24465 9275 + 2.61657 5141i
3	2.83974 1047 + 3.17562 8100i
4	3.33546 0735 + 3.64617 4376i
5	3.76900 5567 + 4.06069 7234i
6	4.15899 8400 + 4.43557 1444i
7	4.51631 9400 + 4.78044 7644i
8	4.84797 0309 + 5.10158 8043i
9	5.15876 7908 + 5.40333 2643i
10	5.45219 2201 + 5.68883 7437i

This present note contains in Table I the first ten complex zeros of $f(z) = \int_0^z e^{-u^2}\,du$, namely z_n, for $n = 1, 2, \ldots, 10$, the first zero z_1 being given to $12D$ and the rest to $9D$. All values of z_n are believed to be correct to *within* a unit in the last place given, with the possible exception of z_2 which is believed to be correct to *around* a unit in the 9th decimal. Newton's method in conjunction with the power series for z_1 and in conjunction with the asympototic series for z_n, $n > 1$, was used for calculating all the zeros. For the first few values of z_n, $n > 1$, the ordinary asymptotic series was not sufficiently accurate and a modification previously published by the author (2) was used to obtain the desired number of decimal places.

[1] Diamond Ordnance Fuze Laboratories, Washington, D. C.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

The values of the zeros z_n may be useful for calculating the inverse error function within certain neighborhoods of the z_n's. For studying the inverse function, instead of z considered as a function of $\int_0^z e^{-u^2}\,du$, it appears preferable to consider z as a function of $y = e^{z^2}\int_0^z e^{-u^2}\,du$ (the zeros of y are also equal to z_n), since y satisfies the very simple linear differential equation

$$\frac{dy}{dz} = 1 + 2zy, \tag{1}$$

so that the inverse function $z(y)$ satisfies

$$\frac{dz}{dy} = \frac{1}{1 + 2zy}. \tag{2}$$

From (1) we have

$$y^{(m)} = 2[zy^{(m-1)} + (m-1)y^{(m-2)}], \quad m > 1, \tag{3}$$

from which one obtains the series around the zero z_n, namely

$$y = y_n + y_n'(z - z_n) + \frac{y_n''}{2}(z - z_n)^2 + \cdots. \tag{4'}$$

But in (4') the coefficients y_n, y_n', y_n'', . . . are very simple since $y_n \equiv y(z_n) = 0$, and from (1) and (3) one sees that $y_n' = 1$, $y_n'' = 2z_n$, $y_n''' = 4z_n^2 + 4$, $y_n^{iv} = 8z_n^3 + 20z_n$. Thus (4') is expressible as

$$y = (z - z_n) + z_n(z - z_n)^2 + \tfrac{2}{3}(z_n^2 + 1)(z - z_n)^3 + \frac{z_n}{6}(2z_n^2 + 5)(z - z_n)^4 + \cdots, \tag{4}$$

all the coefficients of $(z - z_n)^m$ having rather simple expressions in terms of the zero z_n. Then (4) can be easily inverted to yield a series of the form

$$z = z_n + a_1 y + a_2 y^2 + \cdots. \tag{5}$$

The series (5) for the inverse function $z(y)$ will be valid within a circle around z_n up to the nearest singularity of $z(y)$. From (2), since z and y are both bounded in the finite plane, the only finitely located singularities of $z(y)$ will occur at the points (infinite in number) y, z, say Y, Z, where $YZ = -\frac{1}{2}$, or in other words, where the following equation is satisfied:

$$Ze^{Z^2}\int_0^Z e^{-u^2}\,du = -\tfrac{1}{2}. \tag{6}$$

There is a pair of such singular points in the z-plane and in the y-plane upon the imaginary axes, namely at $Z = \pm\ 0.92413\ \ 8873i$, $Y = \pm\ 0.54104\ \ 4225i$. At such a singular point Y, Z, near which (5) will be ineffectual for calculating $z(y)$, one has $\frac{dy}{dz} = 0$, $\frac{d^2y}{dz^2} \neq 0$ from (3) and $Y \neq 0$, so that the direct function $y(z)$ is given by

$$y = Y + \frac{1}{2}\frac{d^2y}{dz^2}\Big|_{z=Z}(z - Z)^2 + \frac{1}{6}\frac{d^3y}{dz^3}\Big|_{z=Z}(z - Z)^3 + \cdots. \qquad (7')$$

But by making use of (3) every coefficient $\frac{d^my}{dz^m}\Big|_{z=Z}$ may be given in terms of Y and Z. Also using the relation $YZ = -\frac{1}{2}$, (7′) is seen to be expressible in the simplest manner as

$$y = Y + Y(z - Z)^2 - \tfrac{1}{3}(z - Z)^3 - \tfrac{1}{6}(Z - 3Y)(z - Z)^4 + \cdots. \qquad (7)$$

From (7) it is apparent that the singularity of $z(y)$ at $y = Y$ is merely a second order branch point, and the inversion of (7) leads to the series that expresses z as a regular function of $(y - Y)^{\frac{1}{2}}$, namely

$$z = Z + b_1(y - Y)^{\frac{1}{2}} + b_2(y - Y) + \cdots. \qquad (8)$$

Acknowledgment

In the final checking of Table I the author was assisted by Miss Isabelle Arsham.

REFERENCES

(1) T. LAIBLE, "Höhenkarte des Fehlerintegrals," *Zeits. Angew. Math. Phys.*, 1951, pp. 484–487.

(2) H. E. SALZER, "Coefficients in an Asymptotic Expansion for $\int_a^b e^{P(u)}du$," *Math. Tables and Other Aids to Comput.*, II, No. 16, Oct. 1946, pp. 188–190.

FHP Motor for Machine Tool Applications.—A new totally enclosed fractional horsepower motor, especially designed for use on machine tools, has been announced by the General Electric Company's General Purpose Component Motor Department. The new heavy duty motor is factory lubricated for ten years' service under normal machine tool conditions. Such lubrication is made possible by a special motor grease, which is highly resistant to oxidation and moisture, they said.

Ideally suited to plugging service and dirty atmospheres, the motor is totally enclosed and fan cooled. The welded-on steel base and ball bearing design help protect the motor against shock and vibration, and maintain accurate motor alignment.

Built to meet the demands for frequent start-stop service, the motor has normal torque for starting, high pull-up torque for inertia loads, and high breakdown torque for peak loads, the engineers said. Insulation materials of Mylar* polyester film, Formex ** wire, and Glyptal** varnish assure resistance to heat, aging, moisture, and electrical stresses.

In horsepower ratings of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$, the polyphase TEFC motor is available with speeds of 1725/1425 and 1140/950 rpm. at 60/50 cycles, and in voltages of 208–220/440, 220/380, and 550.

First USAF "SMART" Test.—The United States Air Force has announced that the first rocket test-sled run on the new USAF supersonic test sled track located at Hurricane Mesa, near Zion National Park, Utah, has been successfully completed.

This newest of the Air Force's rocket-propelled test-track facilities—called SMART (for Supersonic Military Air Research Track)—was designed and constructed by Coleman Engineering Company, Inc., of Los Angeles, for the USAF's Air Research and Development Command.

Prime purpose of the new test facility, on which construction and initial test runs have been completed, is to assist in solving the problem of emergency escape from aircraft traveling at supersonic speeds, for the Aircraft Laboratory of ARDC's Wright Air Development Center.

The SMART high speed track will duplicate, on the ground, actual supersonic flight conditions so that accurate observations can be made under controlled conditions, in the testing of aircraft seat ejection equipment, various types of parachutes, together with the effects of bail-out at supersonic speeds on both equipment and personnel.

Unlike other existing test-track facilities, the SMART project was designed to provide free-flight for test equipment and its recovery for further test use. This is accomplished under the SMART program by catapaulting the test equipment over a 1500-ft. cliff and lowering it by parachute to the desert floor, where it is recoverable.

Full-scale testing of ejection equipment from aircraft at very high speeds is becoming impractical and expensive. By utilizing this controlled, ground-mounted track, equipped with recording equipment, more detailed and accurate information can be obtained, and at less cost.

The ultimate objective of SMART and other Air Force track test projects, is to safeguard the lives of aircrew personnel forced to jump from high-speed aircraft because of inflight emergencies.

* DuPont Trade-mark.

** Reg. Trade-mark of General Electric Company.

ON ELECTROMAGNETIC INDUCTION

BY

PARRY MOON [1] AND DOMINA EBERLE SPENCER [2]

"There seems always to have been an element of mystery connected with the phenomenon [homopolar induction], and an element of doubt on the part of many physicists as to the power of electromagnetic theory to solve the problem uniquely. . . . In the history of the development of the subject there has been a singular freedom from differences of opinion as to the experimental results, but at the same time singular lack of agreement as to the way these results were to be interpreted."

J. T. Tate (1)[3]

"It cannot be urged that it has been shown experimentally that moving circuits and changing currents are rigorously equivalent as regards induced electromotive forces. . . . It is very easy to let the notation carry the burden of the argument . . . and to hold that the value of curl **E** is related to the rate of change of **B** in every case in the way stated by the last equation [curl $\mathbf{E} = -\partial\mathbf{B}/\partial t$]. It is important to point out, however, that by so doing one may be overlooking something of fundamental physical significance."

M. Mason and W. Weaver (2)

1. INTRODUCTION

A perennial subject for discussion in electrical engineering is the generation of an electromotive force by cutting of flux lines or by a time-varying magnetic field. Practical equations for the calculation of the generated emf are

$$V = Blv, \tag{1}$$

$$V = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathfrak{A}, \tag{2}$$

$$V = -\frac{\partial}{\partial t}\int \mathbf{B} \cdot d\mathfrak{A}. \tag{3}$$

The first equation expresses the voltage generated in a conductor of length l, cutting normally through a magnetic field **B**. The velocity of the conductor is **v** with respect to the reference frame in which **B** is measured. Equations 2 and 3 indicate the emf produced by a time-variation of magnetic flux. If the contour that bounds the area is fixed, Eqs. 2 and 3 become identical and $V = -\partial\Phi/\partial t$. But in the

[1] Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[2] Department of Mathematics, University of Connecticut, Storrs, Conn.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

general case where the contour changes with time, the two equations are distinct and must be considered separately.

Discussion usually centers about two questions:

(*a*) Are Eqs. 1 and 2 really independent, or can one be derived from the other?

(*b*) Which equation is the more fundamental?

Since Eq. 2 is derivable directly from one of Maxwell's equations, and since Maxwell's equations are usually considered infallible, Eq. 2 is often regarded as basic, with Eq. 1 as a special case. On the other hand, consideration of the homopolar generator has led several investigators to the conclusion that Eq. 1 is fundamental and that Maxwell's equations are incomplete.

The subject has been discussed ever since the time of Faraday. For instance, an extensive study was made by Lecher (3) in 1895. Some paradoxes were presented by Hering (4) in 1908. Blondel (5) and others argued about the subject in 1915. Another discussion occurred in the *Journal of the Institution of Electrical Engineers* (6) in 1936. Cohn, Bewley, and Slepian (7) exchanged some rapier thrusts in *Electrical Engineering*, 1949 and 1950. Various other studies have been made by physicists (8). In each case, interest seems to rise anew to a sharp peak, after which it gradually declines, leaving the subject much as it was before.

The violence of the controversies and the complete lack of agreement, at the end as well as at the beginning, indicate that the usual treatments of electromagnetic induction must be far from satisfactory. The purpose of the present paper is to show that all ambiguities disappear if we abandon the concept of the magnetic field and handle the entire subject on the basis of inter-particle forces (9).

2. CUSTOMARY THEORY

More fundamental than Eqs. 1 to 3 are the underlying vector point-relations. The basic relation for flux-cutting is

$$\mathbf{E} = \mathbf{v} \times \mathbf{B}, \tag{4}$$

where $\mathbf{E}$ is the electric field strength produced by the motion of a charged particle whose velocity is $\mathbf{v}$ with respect to the magnetic field. From Eq. 4,

$$V = \int_0^l \mathbf{E} \cdot d\mathbf{s} = \int_0^l \mathbf{v} \times \mathbf{B} \cdot d\mathbf{s}. \tag{5}$$

If $\mathbf{v}$, $\mathbf{B}$, and $d\mathbf{s}$ are mutually orthogonal, Eq. 5 reduces to Eq. 1.

Equation 2 comes directly from Maxwell's equation,

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \tag{6}$$

Integrating both sides over an area $\mathfrak{a}(t)$ whose boundary $L(t)$ may vary with time, we obtain at any instant,

$$\int \text{curl } \mathbf{E} \cdot d\mathfrak{A} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathfrak{A}.$$

By Stokes' theorem,

$$\int \text{curl } \mathbf{E} \cdot d\mathfrak{A} = \oint \mathbf{E} \cdot d\mathbf{s} = V,$$

which gives Eq. 2. So Maxwell's equations lead to Eq. 2, not to Eq. 3.

Now consider Eq. 3. If the contour depends on time (10),

$$V = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathfrak{A} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathfrak{A} + \oint \mathbf{v}_L \times \mathbf{B} \cdot d\mathbf{s}, \tag{7}$$

where $\mathbf{v}_L$ is the velocity of the element $d\mathbf{s}$ of the contour. Thus the generated emf may be considered as made of two components; one produced by the time-variation of $\mathbf{B}$ over area $\mathfrak{a}$, the other produced by the motion of the contour L. Note that L is arbitrary: it is merely a path of integration which we have chosen. It may represent a material wire or it may be an imaginary path in space.

Still another equation—advocated by Cohn (7), Kraus (11), and others—is

$$V = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathfrak{A} + \oint \mathbf{v}_m \times \mathbf{B} \cdot d\mathbf{s}. \tag{8}$$

Here the velocity $\mathbf{v}_L$ of the arbitrary contour has been replaced by $\mathbf{v}_m$, the velocity of moving matter. Equation 8 seems to give correct answers, but it is purely *ad hoc* and violates Maxwell's equation 6.

Attempts have been made also to apply Einstein's special theory of relativity. This theory applies to two observers in uniform rectilinear motion, the only means of communication between the two being light signals. Evidently, the peculiar conditions postulated in special relativity are not ordinarily encountered in induced-emf experiments. Generally the rotor of a generator is not in unaccelerated translation with respect to the stator. And the connection between moving and stationary systems is of a more material nature than that effected by light signals. One concludes (12) that relativity has no relevance in the general subject of electromagnetic induction.

3. THE NEW ELECTRODYNAMICS

The pre-Maxwellian approach to electrodynamics is based on an extension of Coulomb's equation. A revival of these ideas (9) replaces

the four equations of Maxwell by a single equation of utmost generality:

$$\mathbf{F}/Q_2 = \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2\left[1 - \frac{3}{2}\cos^2\theta\right] - \mathbf{a}_a \frac{Q_1}{4\pi\epsilon c^2 r}\frac{d}{dt}v(t - r/c) - \mathbf{a}_r \frac{1}{4\pi\epsilon}\frac{\partial}{\partial r}\left[\frac{1}{r}Q_1(t - r/c)\right]. \quad (9)$$

Equation 9 expresses the force on charge Q_2, produced by Q_1 (Fig. 1). The relative velocity of Q_1 with respect to Q_2 is $\mathbf{v}$. In electromagnetic induction, the third term is zero and the distances are usually small enough so that retardation can be neglected. Then Eq. 9 becomes

$$\mathfrak{F}_2 = \mathbf{F}/Q_2 = \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2\left[1 - \frac{3}{2}\cos^2\theta\right] - \mathbf{a}_a \frac{Q_1}{4\pi\epsilon c^2 r}\frac{dv}{dt}. \quad (9a)$$

According to Eq. 9*a*, the induced electric field strength at any point is produced by two distinct causes. The first term of the equation represents the effect of *velocity* of charges; the second term represents

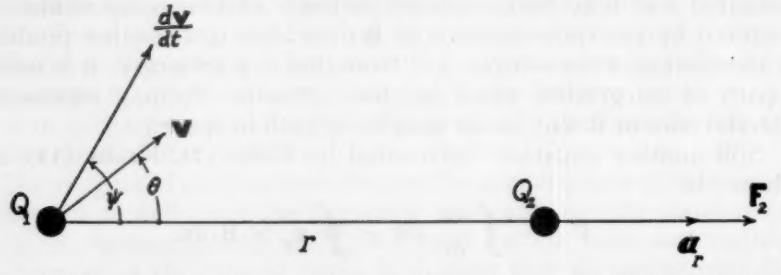


FIG. 1. The new electrodynamics is based on the force between two charges Q_1 and Q_2 with relative velocity $\mathbf{v}$ and acceleration $d\mathbf{v}/dt$.

the effect of *acceleration* of charges. The flux-cutting emf is given by the first term, while the transformer emf is given by the second term. Equation 9*a* offers definite answers to the questions of Section 1. The answer to Question (*a*) is that Eq. 1 cannot be derived from Eq. 2 nor can Eq. 2 be derived from Eq. 1. The two phenomena are separate and distinct. On this basis, Question (*b*) is meaningless.

In the following sections, we shall consider five simple examples of electromagnetic induction. Each case is handled by the usual methods, and these results are compared with the unambiguous results of the new electrodynamics.

4. FIRST EXAMPLE

Consider a conductor sliding at uniform velocity $\mathbf{v}$ through the magnetic field produced by a steady current I (Fig. 2). The current is in a long, straight conductor on the x-axis. An electrostatic voltmeter V indicates the generated voltage.

The magnetic flux density at a point where $y > 0$ is

$$\mathbf{B} = \mathbf{a}_z \frac{I}{2\pi\epsilon c^2 y}. \tag{10}$$

On the basis of flux-cutting,

$$\mathbf{E} = \mathbf{v} \times \mathbf{B} = -\mathbf{a}_y \frac{Iv}{2\pi\epsilon c^2 y}. \tag{11}$$

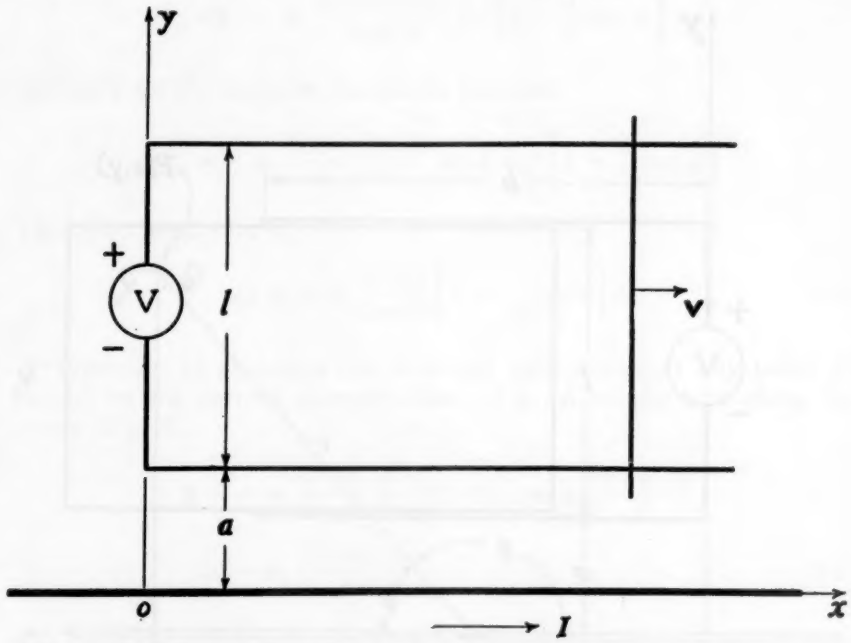


FIG. 2. Simple d-c. generator. The vertical wire moves at uniform velocity v through a magnetic field produced by the current I in a long wire.

Thus the generated emf, in accordance with Eq. 1, is

$$V = \int_a^{a+l} \mathbf{E} \cdot d\mathbf{s} = -\frac{Iv}{2\pi\epsilon c^2} \int_a^{a+l} \frac{dy}{y} = -\frac{Iv}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right), \tag{12}$$

where the direction of V is indicated by the + and − signs of Fig. 2.

Starting with Maxwell's equation 6 or with Eq. 2, we obtain the definite result:

$$V = 0,$$

since $\mathbf{B}$ = const. Thus the prediction of Eq. 2 is not in accord with experiment. Equation 3 or its equivalent, Eq. 7, gives the sum of the

previous values, or

$$V = -\frac{Iv}{2\pi\epsilon c^2}\ln\left(\frac{a+l}{a}\right). \tag{12}$$

Thus Eqs. 1 and 3 give the correct result for this example, while Eq. 2 leads to an incorrect prediction.

Now consider Fig. 2 from the viewpoint of the new electrodynamics. At any point P (Fig. 3), we place a test charge Q_2 which moves at

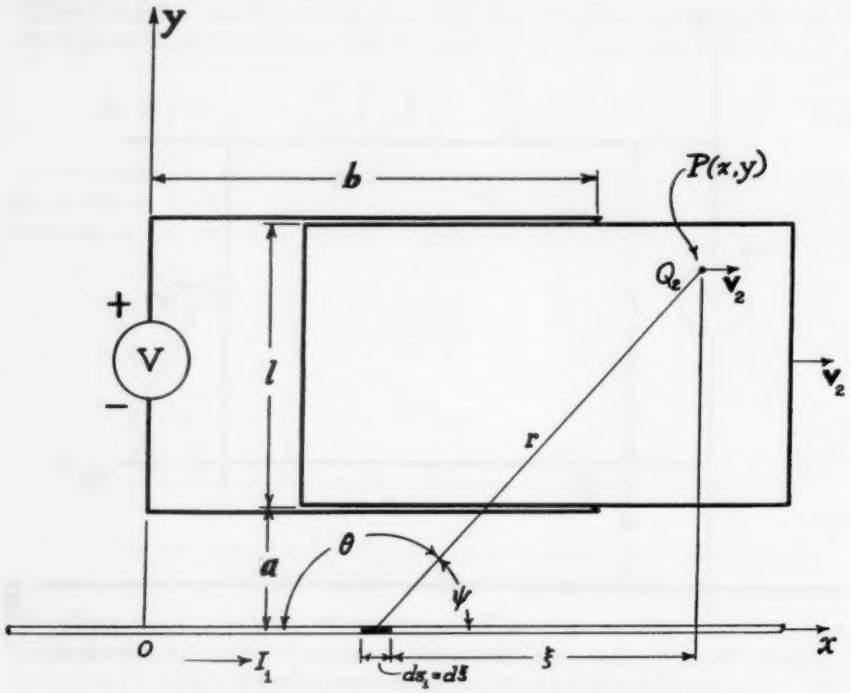


FIG. 3. An electric field is produced at $P(x,y)$ because of the relative motion of the test charge Q_2 and the charges in the current element $I_1 ds_1$.

constant velocity v_2 in the positive x-direction. Since there is no acceleration, the second term of Eq. 9a is zero. The current in the wire is

$$|I_1| = N_1 A_1 |Q_e| v_1,$$

where

N_1 = number of free electrons per cubic meter,
A_1 = cross-sectional area of wire (m^2),
$|Q_e|$ = value of electronic charge (coulomb),
v_1 = drift velocity of electrons (m sec^{-1}).

The field strength at P, caused by current in element $d\mathbf{s}_1$, is the sum of the effects of positive and negative charges in $d\mathbf{s}_1$. The positive charge is

$$dQ_+ = + N_1 A_1 |Q_e| d\mathbf{s}_1,$$

and its velocity with respect to Q_2 is $-\mathbf{v}_2$. Thus Eq. 9*a*, with $\theta = \pi - \psi$ yields

$$d\mathfrak{F}_+ = \mathbf{a}_r \frac{N_1 A_1 |Q_e| d\xi}{4\pi\epsilon c^2 r^2} v_2^2 \left[1 - \frac{3}{2}\cos^2\psi\right].$$

Similarly for the negative charges in the wire,

$$d\mathfrak{F}_- = - \mathbf{a}_r \frac{N_1 A_1 |Q_e| d\xi}{4\pi\epsilon c^2 r^2} (v_1 + v_2)^2 \left[1 - \frac{3}{2}\cos^2\psi\right].$$

Therefore, with $v_1 \ll v_2$,

$$d\mathfrak{F} \cong - \mathbf{a}_r \frac{|I_1| v_2}{2\pi\epsilon c^2 r^2} \left[1 - \frac{3}{2}\cos^2\psi\right] d\xi. \tag{13}$$

Equation 13 expresses the force per unit charge at any point P, caused by the current element $I_1 d\mathbf{s}_1$. For an infinite wire along the x-axis (Fig. 3),

$$\mathfrak{F} = - \mathbf{a}_y \frac{|I_1| v_2}{2\pi\epsilon c^2} \int_{-\infty}^{\infty} \left[1 - \frac{3}{2}\cos^2\psi\right] \frac{\sin\psi\, d\xi}{r^2}$$

$$= - \mathbf{a}_y \frac{|I_1| v_2}{2\pi\epsilon c^2 y}. \tag{14}$$

The total generated emf read by voltmeter V (Fig. 2) is

$$V = - \frac{|I_1| v_2}{2\pi\epsilon c^2} \int_a^{a+l} \frac{dy}{y} = - \frac{|I_1| v_2}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right), \tag{12}$$

which agrees with the result obtained by use of Eqs. 1 and 3.

Equation 12 was obtained by neglecting the drift velocity $\mathbf{v}_1$ of the electrons in comparison with the translational velocity $\mathbf{v}_2$ of the wire. The result is always valid: even in the unusual case that $\mathbf{v}_2 < \mathbf{v}_1$, the effect of $\mathbf{v}_1$ cancels around the complete circuit.

5. SECOND EXAMPLE

Now replace the wire by a metal plate (Fig. 4) that moves to the right at uniform velocity $\mathbf{v}$. On the basis of Eq. 1, it is not quite clear how one convinces himself that a moving plate is identical with a

moving wire. This identification seems particularly troublesome if the plate is replaced by a continuous band or by a cylinder, where the entire apparatus appears the same irrespective of movement. Assuming such an identity, however, one obtains from Eq. 1 exactly the same emf as was obtained for the wire of Fig. 2. This result, of course, agrees with experiment.

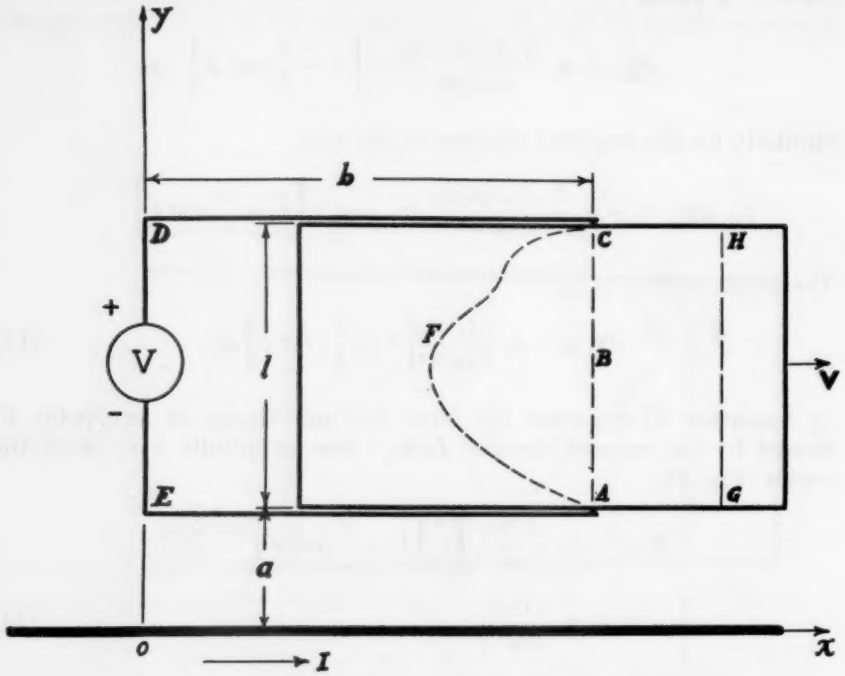


FIG. 4. Unipolar generator. The electrostatic voltmeter V indicates the emf produced by motion of the metal plate.

Starting with Maxwell's equation, however, one may integrate over *any fixed area* such as that enclosed by the dotted line ABC and the wire $CDEA$, Fig. 4. The result is

$$V = -\frac{d\Phi}{dt}. \tag{15}$$

Because the arbitrarily chosen boundary is fixed, Eqs. 2, 3, and 15 are identical and all of them predict zero emf. Considerable effort has been expended by various writers in developing *ad hoc* restrictions and ammendations (13), the purpose of which was to obtain the correct result from Eq. 2 or 3. But none of the suggested changes seems very convincing.

On the other hand, the new electrodynamics shows that every electron in the plate experiences a force in the positive y-direction, producing a difference in potential between the edge $y = a$ and the edge $y = a + l$. The force on the electrons causes a redistribution of charge, with equipotentials $y = \text{const}$ completely across the plate. There are no eddy currents. An electrostatic voltmeter connected anywhere between top and bottom edges will read the voltage given by Eq. 12, irrespective of whether the contacts are opposite each other or not.

6. THIRD EXAMPLE

We now consider an arrangement similar to that of Fig. 2 but with alternating current:

$$I(t) = I_0 \cos \omega t.$$

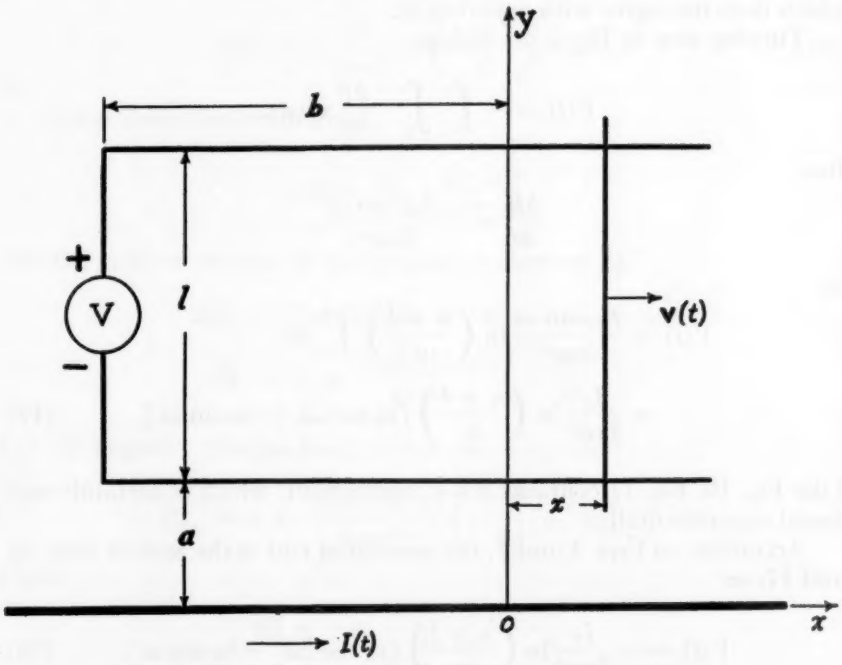


FIG. 5. Oscillating wire in an alternating magnetic field caused by a.c. in the horizontal wire.

A periodic motion is given to the sliding wire (Fig. 5) so that

$$x(t) = \frac{v_0}{\omega} \sin \omega t$$

or

$$v(t) = v_0 \cos \omega t.$$

As in Section 4,

$$\mathbf{B}(t) = \mathbf{a}_z \frac{I(t)}{2\pi\epsilon c^2 y}$$

and, if we accept Eq. 1,

$$\mathbf{E}(t) = \mathbf{v}(t) \times \mathbf{B}(t)$$

$$= -\mathbf{a}_y \frac{I_0 v_0}{2\pi\epsilon c^2 y} \cos^2 \omega t.$$

Thus the generated emf is

$$V(t) = -\frac{I_0 v_0}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right) \cos^2 \omega t, \tag{16}$$

which does not agree with experiment.

Turning now to Eq. 2, we obtain

$$V(t) = -\int_b^x \int_a^{a+l} \frac{\partial B}{\partial t}\, dy\, dx.$$

But

$$\frac{\partial B}{\partial t} = -\frac{I_0 \omega \sin \omega t}{2\pi\epsilon c^2 y},$$

so

$$V(t) = \frac{I_0 \omega \sin \omega t}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right) \int_{-b}^{x} dx$$

$$= \frac{I_0}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right) [v_0 \sin^2 \omega t + b\omega \sin \omega t]. \tag{17}$$

Like Eq. 16, Eq. 17 contains a d-c. component, which is certainly not found experimentally.

According to Eqs. 3 and 7, the generated emf is the sum of Eqs. 16 and 17, or

$$V(t) = -\frac{I_0}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right) [v_0 \cos 2\omega t - b\omega \sin \omega t]. \tag{18}$$

Or Eq. 15 may be employed. The total flux is

$$\Phi = \int_{-b}^{x} \int_a^{a+l} B\, dy\, dx = \frac{I_0 \cos \omega t}{2\pi\epsilon c^2} \int_a^{a+l} \frac{dy}{y} \int_{-b}^{x} dx$$

$$= \frac{I_0}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right)\left[\frac{v_0}{\omega} \sin \omega t \cos \omega t + b \cos \omega t\right].$$

Thus, from Eq. 15,

$$V(t) = -\frac{d\Phi}{dt} = -\frac{I_0}{2\pi\epsilon c^2}\ln\left(\frac{a+l}{a}\right)[v_0\cos 2\omega t - b\omega\sin\omega t]. \quad (18)$$

Now consider the problem on the basis of the new electrodynamics. Since both velocity and acceleration are involved, the first and second terms of Eq. 9 are required. The velocity term is exactly as in Eq. 14 except that I_1 and v_2 are now functions of time:

$$\mathfrak{F}(t) = -\mathbf{a}_y\frac{I_1(t)\cdot v_2(t)}{2\pi\epsilon c^2 y}.$$

Thus the first term of Eq. 9 yields an emf,

$$V(t) = -\frac{I_0 v_0}{2\pi\epsilon c^2}\ln\left(\frac{a+l}{a}\right)\cdot\cos^2\omega t. \quad (19)$$

The acceleration term gives

$$\mathfrak{F} = -\mathbf{a}_a\frac{Q_1}{4\pi\epsilon c^2 r}\frac{dv}{dt}.$$

For the positive charges in the conductor element $d\xi$,

$$dQ_+ = N_1A_1|Q_e|d\xi, \qquad v = -v_2, \qquad \text{and}$$

$$d\mathfrak{F}_+ = -\mathbf{a}_a\frac{N_1A_1|Q_e|d\xi}{4\pi\epsilon c^2 r}\left(-\frac{dv_2}{dt}\right).$$

For the negative charges in $d\xi$, $v = -(v_2 - v_1)$ and

$$d\mathfrak{F}_- = +\mathbf{a}_a\frac{N_1A_1|Q_e|d\xi}{4\pi\epsilon c^2 r}\left(-\frac{d}{dt}[v_2 + v_1]\right).$$

Thus,

$$d\mathfrak{F} = -\mathbf{a}_a\frac{N_1A_1|Q_e|d\xi}{4\pi\epsilon c^2 r}\frac{dv_1}{dt}.$$

But

$$\frac{dI_1}{dt} = N_1A_1|Q_e|\frac{dv_1}{dt} = -I_0\omega\sin\omega t,$$

so the field produced by the current element $I_1\,d\xi$ is

$$d\mathfrak{F} = \mathbf{a}_a\frac{I_0\omega\,d\xi}{4\pi\epsilon c^2 r}\sin\omega t.$$

The field produced by the entire conductor is

$$\mathfrak{F} = \mathbf{a}_a \frac{I_0\omega \sin \omega t}{4\pi\epsilon c^2} \int_{-\infty}^{\infty} \frac{d\xi}{(\xi^2 + y^2)^{\frac{1}{2}}}.$$

Since $\mathbf{a}_a$ is perpendicular to dy, the emf is induced only in the horizontal portions of the loop, and

$$V(t) = [b+x(t)][\mathfrak{F}(t)_{y=a} - \mathfrak{F}(t)_{y=l+a}]$$

$$= \frac{I_0\omega \sin \omega t}{4\pi\epsilon c^2}\left[b + \frac{v_0}{\omega}\sin \omega t\right]\left\{\int_{-\infty}^{\infty} \frac{d\xi}{(\xi^2+a^2)^{\frac{1}{2}}} - \int_{-\infty}^{\infty} \frac{d\xi}{(\xi^2+(a+l)^2)^{\frac{1}{2}}}\right\}$$

$$= \frac{I_0}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right)[v_0 \sin^2 \omega t + b\omega \sin \omega t]. \tag{20}$$

According to Eq. 9*a*, the total emf is the sum of the velocity and the acceleration components, Eqs. 19 and 20, so

$$V(t) = -\frac{I_0}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right)[v_0 \cos 2\omega t - b\omega \sin \omega t], \tag{18}$$

which agrees with the result obtained from Eqs. 2 and 3.

7. FOURTH EXAMPLE

The moving wire of Section 6 is replaced by a moving plate:

$$v(t) = v_0 \cos \omega t,$$

$$\mathbf{B}(t) = \mathbf{a}_z \frac{I_0 \cos \omega t}{2\pi\epsilon c^2 y}.$$

Then, according to the flux-cutting relation, Eq. 1, the generated emf is the same as in Section 6, or

$$V(t) = -\frac{I_0 v_0}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right)\cos^2 \omega t. \tag{16}$$

Using Eq. 2, we have

$$\frac{\partial B}{\partial t} = -\frac{I_0\omega \sin \omega t}{2\pi\epsilon c^2 y}$$

and

$$V(t) = \frac{I_0 b\omega}{2\pi\epsilon c^2} \ln\left(\frac{a+l}{a}\right)\sin \omega t. \tag{21}$$

Since the boundary *ABCDEA* is fixed, Eq. 3 also gives Eq. 21. Both Eqs. 16 and 21 are incorrect, so all the classical results are wrong.

The new electrodynamics, on the other hand, gives exactly the same result for the moving plate as for the moving wire. In either case, the generated emf is expressed by Eq. 18, assuming that the horizontal wires to the voltmeter are spaced by distance l. In the more general case where the spacing is not l, the coefficient of the single-frequency term will be altered, as it should be.

8. FIFTH EXAMPLE

If there is a time-variation of flux and no relative motion between circuits, we have the case of pure transformer action, and Eq. 2 applies without question. Some writers have attempted to explain this case as an example of flux cutting, the assumption being that the flux originates in the primary conductor and spreads out into space, cutting the secondary conductors and thus inducing an emf. That such an explanation is untenable is seen by visualizing a toroid with two windings. Here the flux remains in the core at all times and certainly does not cut the conductors. Thus for fixed circuits, Eqs. 2 and 3 apply and Eq. 1 does not.

Since there is no relative motion between circuits, the first term of Eq. 9*a* is zero and only the acceleration term applies. As shown previously (**9**), the mutual inductances and induced emfs in fixed circuits, obtained by the new electrodynamics, are identical with those calculated by classical methods.

9. CONCLUSIONS

The results of our five examples are tabulated as follows:

Example	Flux-cutting, Eq. 1	Maxwell, Eq. 2	Moving boundary, Eq. 3	New electrodynamics, Eq. 9*a*
I	•	×	•	•
II	•	×	×	•
III	×	×	•	•
IV	×	×	×	•
V	×	•	•	•

A dot indicates a correct answer; a cross indicates an incorrect one. The classical equations sometimes yield the correct result, sometimes not. Even Eq. 3, according to our interpretation, gives a false prediction for the homopolar generator, though in three other examples it happens to be correct.

The table indicates the uncertainty of the classical theory of electromagnetic induction. Knowing the experimental result, any engineer can find the particular equation that gives him the correct answer. And he can always think of a plausible reason for using this equation and not another. But such a procedure is a treacherous basis for

exploring the unknown. The new electrodynamics, on the other hand, has given the correct result, without ambiguity, in all problems to which it has been applied.

REFERENCES

(1) J. T. TATE, "Unipolar Induction," *Nat. Res. Council, Bulletin*, Vol. 4, No. 24, p. 75 (1922).

(2) M. MASON AND W. WEAVER, "The Electromagnetic Field," Chicago, Univ. of Chicago Press, 1929, p. 257.

(3) E. LECHER, "Eine Studie über unipolare Induction," *Ann. d. Phys.*, Vol. 54, p. 276 (1895).

(4) C. HERING, "An Imperfection in the Usual Statement of the Fundamental Law of Electromagnetic Induction," *A.I.E.E. Trans.*, Vol. 27, p. 1341 (1908);
"A New Factor in Induction; the 'Loop' vs the 'Cutting of Lines of Force' Laws," *El. Wld.*, Vol. 51, p. 559 (1908);
"Revision of Some of the Electromagnetic Laws," JOUR. FRANKLIN INST., Vol. 192, p. 599 (1921);
"Electromagnetic Forces; a Search for More Rational Fundamentals," *A.I.E.E. Trans.*, Vol. 42, p. 311 (1923).

(5) A. BLONDEL, "The Laws of Induction," *Electrician*, Vol. 75, p. 344 (1915);
Discussion: *ibid.*, pp. 559, 634, 671, 711, 815, 855, 890, 927, 965; Vol. 76, p. 169.

(6) W. CRAMP AND E. H. NORGROVE, "Some Investigations on the Axial Spin of a Magnet and on the Laws of Electromagnetic Induction," *J. Inst. Elec. Engrs.*, Vol. 78, p. 481 (1936);
Discussion: Vol. 79, p. 344 (1936).

(7) G. I. COHN, "Electromagnetic Induction," *El. Eng.*, Vol. 68, p. 441 (1949);
Discussion: *ibid.*, pp. 645, 678, 762, 878, 985, 1080, 1113; Vol. 69, pp. 187, 481, 1025, 1088, 1138; Vol. 70, pp. 67, 161, 255, 352, 451;
J. SLEPIAN, "Lines of Force in Electric and Magnetic Fields," *Am. J. Phys.*, Vol. 19, p. 87 (1951).

(8) H. POINCARÉ, "Sur l'induction unipolaire," *L'Eclairage Electrique*, Vol. 23, p. 41 (1900);
S. J. BARNETT, "On Electromagnetic Induction and Relative Motion," *Phys. Rev.*, Vol. 35, p. 323 (1912);
"A Report on Electromagnetic Induction," *A.I.E.E. Trans.*, Vol. 38, p. 1495 (1919);
"Electric Fields Due to the Motion of Constant Electromagnetic Systems," *Phil. Mag.*, Vol. 44, p. 1112 (1922);
G. B. PEGRAM, "Unipolar Induction and Electron Theory," *Phys. Rev.*, Vol. 10, p. 591 (1917);
E. G. CULLWICH, "An Experiment on Electromagnetic Induction by Linear Motion," *J. Inst. Elec. Engrs.*, Vol. 85, p. 315 (1939).

(9) P. MOON and D. E. SPENCER, "A New Electrodynamics," JOUR. FRANKLIN INST., Vol. 257, p. 369 (1954);
"Electromagnetism without Magnetism: An Historical Sketch," *Am. J. Phys.*, Vol. 22, p. 120 (1954).

(10) See, for instance, R. BECKER, "Theorie der Elektrizität," Leipzig, B. G. Teubner, 1944, Vol. I, p. 39.

(11) J. D. KRAUS, "Electromagnetics," New York, McGraw-Hill Book Co., 1953, p. 292.

(12) P. MOON AND D. E. SPENCER, "Unipolar Induction and Maxwell's Equations," to be published.

(13) For example, L. V. BEWLEY, "Flux Linkages and Electromagnetic Induction," *A.I.E.E. Trans.*, Vol. 48, p. 327 (1929);
El. Eng., Vol. 68, p. 1113 (1949).

NUCLEAR ENERGY NEWS

AEC DIVISION OF CIVILIAN APPLICATION

The establishment of a Division of Civilian Application in the AEC staff was announced by the Atomic Energy Commission on June 29, 1955.

The new Division takes the place and assumes the responsibilities of the Division of Licensing and carries out other civilian use activities contemplated by the Atomic Energy Act of 1954 which encourages "widespread participation in the development and utilization of atomic energy for peaceful purposes." Its first Director is Harold L. Price, formerly Director of the Division of Licensing.

The Division is responsible for developing AEC regulations and other instructions dealing with licenses, authorizations, and other activities related to civilian use, and providing for the issuance of these regulations and instructions; administering the AEC licensing program; developing pricing schedules for materials and services made available to licensees and prices to be paid by the AEC for materials produced or returned by licensees, and providing for the issuance of these schedules; authorizing the distribution of source, special nuclear, by-product and other materials for licensed use; administering the AEC's program of access to restricted data for civilian use; and handling authorizations to export equipment and materials or to engage in atomic energy activities outside the United States.

In establishing the Division of Civilian Application, the AEC has transferred to it the following staff functions previously assigned to other divisions: the activities of the Licensing Controls Branch, Division of Construction and Supply, and certain functions of the now abolished Industrial Liaison Branch of the Division of Reactor Development.

AEC REGULATION FOR LICENSING OPERATORS

The Atomic Energy Commission has approved a proposed regulation establishing the procedures and criteria for the issuance of licenses to operators of licensed production and utilization facilities, including nuclear reactors.

Under the regulation a license will be granted to a person who manipulates the controls of a licensed facility. A license will be required only for a person who has the responsibility for making decisions as to when or how to manipulate the controls, and either manipulates the controls himself or directs another person to do so in

his presence. This provision would allow unlicensed students or trainees to handle the controls under the personal direction of a licensed operator.

The regulation is further limited to apply only to controls which might, if improperly handled, result in the release of dangerous amounts of atomic energy or radioactive material. The applicant must pass a medical examination designed to show that "the physical condition and the general health of the applicant are not such as to be expected to cause operational errors which might endanger public health and safety." The license will be subject to renewal at intervals of two years, with a repetition of the medical examination.

The applicant will also be required to pass an operating test and a written examination. They are to test "the applicant's understanding of and familiarity with the following aspects of the facility: (1) the general design and operating characteristics; (2) the control and safety mechanisms; (3) all control-station instrumentation; (4) standard operating procedures; (5) emergency shut-down system; and (6) such other aspects as may be important to the safe operation of the facility."

RADIOISOTOPE PRODUCTION AT HANFORD

Production of radioactive isotopes at the Hanford plutonium plant near Richland, Washington is a sizeable contribution to the present peacetime use of atomic energy, according to a recent announcement by the general manager of the plant, which is operated by the General Electric Company for the Atomic Energy Commission. The Hanford plant produces a large share of the nation's supply of radioactive isotopes for peacetime use, while at the same time performing its prime function of producing plutonium, a man-made element now used mainly in atomic weapons.

These radioisotopes are made by placing chemical elements such as cobalt, iron or nitrogen inside atomic reactors where rapidly moving neutrons convert them to radioactive forms of the same or another element.

Hanford produces radioisotopes at the request of Oak Ridge National Laboratory (ORNL). Oak Ridge is the sole supplier of *processed* radioisotopes and the point of sale or authority for sale for all the AEC's radioisotopes. ORNL purifies, packages and stores isotopes produced at its own site, at Hanford and other reactor sites. From Oak Ridge they are shipped to customers in all parts of the United States and are available to 53 foreign countries.

Hanford plays a major role in the production of Carbon-14, one of the radioisotopes widely used in industry, research, agriculture and medicine. Nitrogen-containing compounds are irradiated in the Hanford nuclear reactors where radioactive-carbon is formed. These irradiated materials are sent to Oak Ridge where the Carbon-14 is

separated and purified. This combined effort of Hanford and Oak Ridge produces all the radioactive carbon in the United States.

EXHIBITS AT GENEVA CONFERENCE

Some 100 U. S. industrial and commercial firms, along with 50 academic, professional and private educational and research organizations, were represented in exhibits at the International "atoms-for-peace" conference at Geneva, Aug. 8–20.

Eighty-three firms and organizations contributed to the official U. S. technical exhibit at the Palais des Nations, site of the United Nations sponsored conference. Limitations on space at the Palais building at the Conference site resulted in a United Nations requirement that all exhibits there be sponsored by participating governments, be non-commercial in character, and be designed primarily for the delegates and technical experts attending the Conference.

Fourteen commercial publishers and approximately 30 scientific, engineering and other professional organizations were represented in a display of more than 100 U. S. periodicals in the nuclear energy field.

Also the AEC contributed a technical reference library of several thousand AEC unclassified reports.

A 10-kilowatt operating research reactor of the "pool" type highlighted the U. S. official exhibit. (See this JOURNAL, Vol. 259, p. 533.) In addition, there were scale models of power and research reactors operating or planned in the United States.

Many additional national and commercial exhibits were displayed at the spacious Exposition Palace in mid-city. They included the "Atom-for-Peace" exhibit of the U. S. Information Service, covering 15,000 sq. ft., parts of which had been contributed by a score of firms and organizations.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS

IMPROVED METHOD FOR APPLYING CERMETS

A rapid, economical process for applying a common type of cermet coating to a metal base is now in use at the National Bureau of Standards. The cermet powder in this case is a chromium-boron-nickel mixture that is combined with a ceramic frit. The process employs ordinary ceramic coating procedures to replace the former flame-spraying method and provides a hard, high-temperature-resistant facing for metal parts. Developed by D. G. Moore and J. R. Cuthill of the Bureau's enameled metals laboratory, the coatings have excellent thermal shock resistance and are reasonably ductile.

Cermets possess properties of both metals and ceramics. The chromium-boron-nickel types have some degree of ductility, are resistant to thermal shock and oxidation at high temperatures, and retain their strength and hardness at high temperatures as well. In recent years, chromium-boron-nickel layers have been used extensively as hard facings for metal parts subjected to the severe conditions of erosion and oxidation. In one commonly-used process, the chromium-boron-nickel powder is flame-sprayed onto the metal part. This overlay is then fused to the base metal with an oxy-acetylene torch. The resulting layer is dense and well-bonded but has several disadvantages. First, it tends to be expensive for large-scale production; second, the flame-spray technique is unsuited to the coating of cylindrical parts of small inside diameter; third, the process is not feasible where thin layers—about 0.002 in.—are required. To overcome these objections, the Bureau undertook a study of other methods of applying chromium-boron-nickel cermets. So far, the use of ordinary ceramic coating procedures appears to be the most successful.

In the NBS process, a slip is prepared from a cermet powder and a ceramic frit. However, instead of flame-spraying the material, the usual ceramic coating procedure is followed and the slip is applied to the metal part at room temperature by either dipping or spraying. The part is then fired at a temperature above the melting point of both the cermet and the frit. During the firing the ceramic phase in the coating serves as a flux to permit welding of the cermet particles without the necessity of a highly-purified, oxygen-free atmosphere. Depending on the firing temperature, the ceramic material either "sweats out" to the surface or remains as occlusions in the cermet network, while the cermet particles melt to form a continuous layer that becomes brazed to the surface of the basis metal.

In one series of experiments, coatings with different ratios of frit to cermet powder were prepared and applied to various basis metals. It was discovered that the sweating out of the glassy phase to the surface of the cermet layer occurred at a firing temperature of 1900° F. for coatings which contained 5, 10, or 20 parts by weight of frit. However, a temperature of 2100° F. was required before sweating out occurred with a coating which contained 50 parts by weight of frit. In the cases where sweating out did not occur at all, the glassy phase was found to be enmeshed in a welded network of the cermet material.

Firing time had little effect on the appearance of the coatings; a coating appears the same after 16 mins. of firing as it does after 2 mins. Firing in air gave satisfactory coatings on alloys that initially had good oxidation resistance, but a furnace atmosphere of low oxygen content was necessary to achieve satisfactory protection on such metals as molybdenum, ingot iron, and several low-alloy steels.

Neither thorough cleaning nor roughening of the metal surface prior to applying the coating was essential. Satisfactory coatings were obtained on smooth surfaces which had been only mildly degreased before applying the slip.

The coating with the best continuity of the Cr-B-Ni layer, and with the best appearance, was obtained from a mixture of 10 parts of frit, 90 parts of cermet powder, and 5 parts of clay by weight. This coating was designated M-60. With this mixture, coating thicknesses in the range of 0.004 to 0.008 in. can be achieved by single-coat application. For thicknesses greater than this range, multiple coatings are recommended. For thinner coatings, finer grinding of the slip is required. Such thin compositions have been designated M-60F.

With the coarsely ground coating (M-60), the glassy phase forms at the surface of the cermet as small beads when fired according to the recommended procedure. These beads tend to flake off on cooling, but light sandblasting or treating in molten alkali will result in a specimen of more acceptable appearance. On the other hand, when the finely-ground coating (M-60F) is used, and application thicknesses are less than 0.003 in., the glassy phase that comes to the surface forms a uniform glaze that adheres to the underlying cermet layer. In this case, the specimen appears to be ceramic coated.

The possible use of the M-60F coating for protecting low-alloy steels against oxidation at 1500° F. was also investigated. Uncoated specimens scaled rapidly, whereas the M-60F coating prevented any loss in effective thickness for the maximum test period of 800 hr. However, spectrochemical analysis shows that boron diffuses into the alloy structure during the heat treatments. In one test, the boron content of a low-alloy steel reached 0.028 per cent by weight after the 800-hr. treatment at 1500° F.

COLORIMETER FOR PYROTECHNIC SMOKES

Pyrotechnic smokes of various colors are used on the battlefield for ground-to-air signals to prevent inadvertent attacks by air forces on friendly troops. These smokes must be vividly colored to distinguish them from the terrain and to minimize confusion with battle smokes. To aid in the production of more vivid smoke-colors and to determine whether purchased smokes meet color specifications, the National Bureau of Standards has developed an accurate, portable photoelectric colorimeter. The instrument was designed and constructed by I. Nimeroff and S. W. Wilson of the Bureau's photometry and colorimetry laboratory[1] under the sponsorship of the Army Chemical Corps.

The NBS colorimeter uses a system of color measurement adopted in 1931 by the Commission Internationale de l'Eclairage (CIE). This system reduces spectrophotometric data to color coordinates in three-dimensional space. Colors are expressed in terms of three functions which correspond roughly to the three primary colors—red, green, and blue—and are designated $\bar{x}$, $\bar{y}$, and $\bar{z}$, respectively. The CIE system was chosen for use in the Bureau's instrument because it correlates closely with the color response of an average normal observer and can be readily converted to other color systems.

The colorimeter consists essentially of a lens and three phototube-filter combinations mounted in a brass tube. A specially built 36-in. chamber, with a large window in one side, contains the colored smoke to be measured. Four 150-watt flood lamps uniformly illuminate the chamber window. A blower is installed in the chamber to stir the smoke for uniformity and for exhaust purposes after an analysis.

In making a color analysis, the light from a central area of the chamber window is focussed by the lens onto the three phototube-filter combinations. Electrical outputs of the phototubes are indicated on three photometers, showing the intensities of the red, green, and blue components of the smoke.

The multiplier phototubes view the chamber through filters designed to approximate the three daylight CIE standard observer functions. As the CIE $\bar{y}$ and $\bar{z}$-functions are unimodal, filter design is easily accomplished for these two functions. However, to approximate the two lobes of the bimodal $\bar{x}$-function, it was necessary to design a special divided filter.

Earlier techniques for approximation of the $\bar{x}$-function have used either numerical or electrical addition of a portion of the $\bar{z}$-function to the long-wave lobe of the $\bar{x}$-function. In the NBS colorimeter, this approximation is accurately made by using a filter divided into two sectors. One sector has spectral transmittance approximating the

[1] For technical details see, "A Colorimeter for Pyrotechnic Smokes," by I. Nimeroff and S. W. Wilson, *J. Research NBS*, Vol. 52, p. 195 (1954).

long-wave (red) lobe; the other has spectral transmittance approximating the short-wave (blue) lobe of the $\bar{x}$-function.

Measurements made on the colorimeter have been compared with visual estimates of the colors in terms of Munsell color charts. The results indicated that the CIE tristimulus standard observer functions have been adequately approximated, and that edge effects and rear wall reflection effects in the chamber are negligible.

With some slight modification, the NBS colorimeter should be useful in other types of color evaluation. For example, the transmitted color of a translucent material might be measured by mounting the illuminating lamps beyond the chamber and turning them to face the photocells with the material interposed. The colorimeter could also be used to investigate the color fidelity of a color television tube. In this use, the tube under investigation would replace the chamber, and the illuminating lamps would not be necessary.

THE FRANKLIN INSTITUTE

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A. M. until 5 P. M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; and 2 P. M. until 10 P. M. on Wednesdays.

407 items were added this month.

RECENT ADDITIONS

AERONAUTICS

FRENTZEN, AUGUST. Der Weltenflug. 1926.

AGRICULTURE

DAVIS, J. G. A Dictionary of Dairying. Ed. 2. 1955.
FOX, SIR CYRIL S. Water. 1952.
VICKERY, HUBERT BRADFORD, ET AL. Chemical Investigations of the Tobacco Plant. 1933.

ARCHITECTURE AND BUILDING

JOHNSTON, A. LONGSTOFF, JR. Steam Piping. 1916.
LANDER, CECIL H. Ventilation and Humidity. 1914.
SVENSEN, CARL L. A Handbook on Piping. Ed. 2. 1922.

ASTRONOMY

FATH, EDWARD ARTHUR. The Elements of Astronomy. Ed. 5. 1955

BIOGRAPHY

BABBAGE, CHARLES. Passages from the Life of a Philosopher. 1864.
TVETERAS, EGIL. To Boktrykkere; Benjamin Franklin, William Morris. 1954.

BIOLOGICAL CHEMISTRY

BACQ, Z. M. AND ALEXANDER, PETER. Fundamentals of Radiobiology. 1955.
CHARGAFF, EDWIN AND DAVIDSON, J. N., ED. The Nucleic Acids. Vol. 1. 1955.

CHEMISTRY AND CHEMICAL TECHNOLOGY

AHRENS, LOUIS HERMAN. Quantitative Spectrochemical Analysis of Silicates. 1954.
BAUER, KARL HUGO AND MOLL, HEINRICH. Die Organische Analyse. 1954.
BROGLIE, PRINCE LOUIS VICTOR DE. Mécanique Ondulatoire et Cinématique Chimique. 1955.
DAUBERT, B. F., ED. Fats, Oils, Detergents Yearbook 1954. 1955.
ELION, E. Recherches sur le Rôle du phosphore dans la Vie de la Levure et dans la Fermentation Alcoolique. 1927.
GETMAN, FREDERICK HUTTON. Physical Chemistry. 1955.
GOULD, EDWIN S. Inorganic Reactions and Structures. 1955.
GUNTHER, FRANCIS A. AND BLINN, ROGER C. Analysis of Insecticides and Acarides. 1955.

HOPFF, HEINRICH; MÜLLER, ALFRED AND WENGER, FRIEDRICH. Die Polyamide. 1954.
KÜHL, HANS. Zementchemie. 1929.
MEITES, LOUIS. Polarographic Techniques. 1955.
PEISER, H. S., ET AL, ED. X-ray Diffraction by Polycrystalline Materials. 1955.
ROBINSON, ROBERT. The Structural Relations of Natural Products. 1955.
TIBBLES, WILLIAM. The Theory of Ions. 1909.

ELECTRICITY AND ELECTRICAL ENGINEERING

BERTHOLD, HORST. Stufenlos Verstellbare Mechanische Getriebe. 1954.
GOTTER, GOTTFRIED. Erwärmung und Kühlung Elektrischer Maschinen. 1954.
GUILBAUD, GEORGES TH. La Cybernétique. 1954.
HERSCHBERGER, W. D. Principles of Communication Systems. 1955.
HICKEY, HENRY V. AND VILLINESS, WILLIAM M. Elements of Electronics. 1955.
LANGSDORF, ALEXANDER SUSS. Theory of Alternating-Current Machinery. Ed. 2. 1955.
NATIONAL PHYSICAL LABORATORY. Precision Electrical Measurements. 1955.
SAY, M. G. Electro-technology. 1955.
SCHMIEDEL, KARL. Die Prüfung der Elektrizitätszähler; Messeinrichtungen; Messmethoden und Schaltungen. Ed. 4. 1954.

ENGINEERING

WHITE, ALFRED H. Engineering Materials. 1939.

GENERAL

The American Peoples Encyclopedia Yearbook. 1953.
WALTON, E. W. KEVIN. Two Years in the Antarctic. 1955.
Woolworth's First 75 Years; the Story of Everybody's Store. 1954.

GEOLOGY

EMMONS, WILLIAM HARVEY, ET AL. Geology. Ed. 4. 1955.
THE UNIVERSITY OF CHICAGO; THE NATIONAL RESEARCH FOUNDATION AND THE NATIONAL SCIENCE FOUNDATION. Proceedings of the Conference on Nuclear Processes in Geologic Settings. 1953.
WAHLSTROM, ERNEST E. Petrographic Mineralogy. 1955.

HOROLOGY

BEILLARD, ALFRED. Recherches sur l'Horlogerie ses Inventions et ses Célébrités. 1895.

HYDRAULICS

BIÉSEL, F., AND OTHERS. Laboratory Wave-Generating Apparatus. 1954.
DEHNERT, HANS. Schleusen und Nebewerke. 1954.
INTERNATIONAL ASSOCIATION FOR HYDRAULIC RESEARCH AND AMERICAN SOCIETY OF CIVIL ENGINEERS. Minnesota International Hydraulics Convention. Proceedings. 1953.

INDUSTRIAL MANAGEMENT

BRAUN, CARL F. Management and Leadership. 1954.

MANUFACTURE

ALLISON, ARCHIBALD. The Manufacture of Chilled Iron Rolls. 1929.
BILLIG, KURT. Precast Concrete. 1955.
BOURRY, EMILE. A Treatise on Ceramic Industries. 1911.
COOK, GEORGE S., ED. Paint Industry Materials. 1955.
GRAHAM, KENNETH A. AND PINKERTON, H. L., ED. Electroplating Engineering Handbook. 1955.
SEARLE, ALFRED B. The Clayworker's Hand-book. Ed. 2. 1911.

MATHEMATICS

BESICOVITCH, ABRAHAM SAMOILOVITCH. Almost Periodic Functions. 1954.
BETH, H. J. E. Inleiding to de Differentiaalen Integraalrekening. Ed. 6. 1953.
CODDINGTON, EARL A. AND LEVINSON, NORMAN. Theory of Ordinary Differential Equations. 1955.
EASTERN JOINT COMPUTERS CONFERENCE. Proceedings 1954. Theme: Design and Application of Small Digital Computers. 1955.
HUNTINGTON, EDWARD VERMILYE. The Continuum and Other Types of Serial Order. Ed. 2. 1955.
YOUNG, J. W. A. Monographs on Topics of Modern Mathematics. 1955.

MECHANICAL ENGINEERING

BARNARD, THEODORE ROBERT. Mechanical Engineering. 1954.
INNES, CHARLES H. The Centrifugal Pump, Turbines and Water Motors. Ed. 5. 1909.
LOEWENSTEIN, LOUIS C. AND CRISSEY, CLARENCE P. Centrifugal Pumps. 1922.
NATIONAL PHYSICAL LABORATORY. Engineering Dimensional Metrology. Vol. 1–2. 1955.
STEPANOFF, A. J. Turboblowers. 1955.

METALLURGY

COTTRELL, A. H. Theoretical Structural Metallurgy. Ed. 2. 1955.
HUYBRECHTS, M. Chimie Analytique Appliquée à la Métallurgie. Ed. 3. 1953.
Iron and Steel; Report of a Productivity Team. 1952.

MINING ENGINEERING

BRIGGS, HENRY. The Ventilation of Mines. 1929.
LISTER, J. E. The Screening and Grading of Materials. 1924.

PHYSICS

BARRET, PIERRE. Température et Équilibre Thermique dans une Flamme de Diffusion. 1952.
BROGLIE, PRINCE LOUIS VICTOR DE, ED. Les Applications de la Mécanique Ondulatoire à l'Étude de la Structure des Molécules. 1953.
BRÜGEL, WERNER. Einführung in die Ultrarotspektroskopie. 1954.
DURBIN, FRANK M. Introduction to Physics. 1955.
KEENAN, JOSEPH H. AND KEYES, FREDERICK G. Thermodynamic Properties of Steam Including Data for the Liquid and Solid Phases. 1944.
NORRIS, EARLE B.; THERKELSEN, ERIC AND TRENT, CLARENCE E. Applied Thermodynamics. Ed. 3. 1955.
PEIERLS, R. E. Quantum Theory of Solids. 1955.
RICHARDSON, K. I. T. The Gyroscope Applied. 1954.
SERRUYS, MAX. Étude Générale de l'Écoulement d'un Gaz. 1952.
Two Monographs on Electrokinetics. 1951.
YOURGRAU, WOLFGANG AND MANDELSTAM, STANLEY. Variational Principles in Dynamics and Quantum Theory. 1955.

PLASTICS

MARK, H.; PROSKAUER, E. S. AND FRILETTE, V. J., ED. Resins, Rubbers, Plastics Yearbook 1954. 1955.

SCIENCE

BELL, WHITFIELD J., JR. Early American Science. 1955.
VERNOTTE, PIERRE. Les Principes Physiques de la Formulation des Lois Experimentales. 1952.

TEXTILES

BURKETT, CHARLES WILLIAM AND POE, CLARENCE HAMILTON. Cotton. 1906.
HARRIS, MILTON AND MARK, H., ED. Natural and Synthetic Fibers Yearbook 1954. 1955.

WOOD

FOOD AND AGRICULTURE ORGANIZATION OF THE UNITED NATIONS. World Forest Resources. 1955.
NATIONAL FARM WOODLOT CONFERENCE. Proceedings. 1953. 1953.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

MEES, C. E. K.: The Structure of the Developed Photographic Image
MISHKIN, E.: On the Theory of a Two Stage A. C. Rotary Amplifier
YU, YI-YUAN: On the Complex Representation of the General Extensional and Flexural Problems of Thin Plates and Their Analogies
MOON, PARRY AND DOMINA EBERLE SPENCER: On the Ampère Force
SWANN, W. F. G.: Present Status of Atmospheric Electricity
MARIN, JOSEPH AND H. A. B. WISEMAN: Triaxial Plastic Stress-Strain Relations for an Aluminum Alloy 14ST–4

TO MEMBERS AND THOSE WHO KNOW THE FRANKLIN INSTITUTE

All of us associated with The Franklin Institute are gratified by the support given by its benefactors in the past. Without their generosity our privately endowed Institute would have been unable to serve industry, the professions, and the public as it has and should.

There is still a continuing and pressing need of contributions and bequests —both for general and specific endowment, and of income to be judiciously used. Your response will be most earnestly appreciated.

Checks should be made payable to The Franklin Institute, and sent to The Franklin Institute, Benjamin Franklin Parkway at 20th Street, Philadelphia 3, Pennsylvania.

Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

We shall be glad to supply any additional information regarding gifts and memorials. Please write to the above address, attention of the Endowment Department.

NOTES FROM THE BARTOL RESEARCH FOUNDATION

W. F. G. SWANN, DIRECTOR

NATURE OF CONDUCTION IN THORIUM OXIDE SINGLE CRYSTALS *

BY

W. E. DANFORTH AND J. H. BODINE

Recent considerations relative to the theory of phenomena in mixed conductors have led us to the view that, in a fresh unpolarized unactivated crystal of thorium oxide, the conduction takes place almost entirely by ions.

The considerations referred to have to do with the analysis of the polarization curve. Letting E_0 be the emf. which appears when a constant current is applied, and E_∞ the final emf. which exists as a result of polarization, it appears that the fraction of current carried by electrons should be, in the limit of small currents, E_0/E_∞. This is found experimentally to be a quantity less than 1 per cent.

The high electronic conductivity observed in polycrystalline thoria aggregates, as used in electron tube cathodes, must be understood as resulting from the presence of electrolytically produced thorium.

* Assisted by the U. S. Office of Ordance Research and the U. S. Navy Bureau of Ships.

INELASTIC SCATTERING OF 14 MEV NEUTRONS *

BY

G. W. McCLURE AND D. W. KENT

The Bartol 100 Kev Cockcroft-Walton accelerator now produces a flux of 2×10^8 14 Mev neutrons per sec via the $H^3(H^2, n)He^4$ reaction. A scintillation counter built into the target chamber acts as an indicator of the absolute neutron yield by counting the He^4 recoils from the target.

Experimentation thus far completed has consisted mainly of inelastic scattering measurements on the Fe nucleus. The γ-ray spectrum resulting from the neutron bombardment of an Fe scattering sample (natural isotope mixture) shows discrete γ-rays at 0.850, 1.25, 1.82 and 2.12 Mev all of which correspond to transitions between known levels in the 92 per cent abundant Fe^{56} isotope. The production cross sections for the two lower energy γ-rays are approximately 0.75 and 0.26 barn, respectively. An investigation of the 2.57-hr. Mn^{56} activity induced in the Fe scatterer by neutron bombardment indicates a cross section of 72 millibarns for the reaction $Fe^{56}(n,p)Mn^{56}$. All of the cross sections listed are accurate to about $\pm$ 10 per cent.

Further inelastic scattering work employing n-γ coincidence measurements are currently in progress.

* Assisted by the U. S. Atomic Energy Commission.

NUCLEAR RESONANCE FLUORESCENCE IN Se^{74} *

BY

F. R. METZGER

Using a source of As^{74} in the form of arsine gas, a resonance fluorescence effect from the 635 kev level in Se^{74} has been observed. Due to the small abundance of Se^{74} (0.87 per cent), the effect is not very pronounced and the mean life of 6×10^{-12} second, calculated from the magnitude of the resonance fluorescence cross section, is therefore uncertain to within a factor of two.

* Assisted by the Joint Program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

RADIATIONS FROM Pt^{195}, Pt^{193}, AND Pt^{191}

BY

V. R. POTNIS,** C. E. MANDEVILLE AND JOHN S. BURLEW ***

Using scintillation spectrometers of NaI(Tl), photopeaks of gamma rays from the 4-day Pt^{195} were detected at 31, 99, and 130 kev. The X-rays of platinum were also in evidence. By a consideration of conversion coefficients and relative intensities of the unconverted radiations, it was concluded that the observed unconverted quantum radiations at 130 kev are emitted in the cross-over transition of the 31 kev-99 kev cascade. A gamma ray of energy 133 kev was observed in the decay of Pt^{193} and was found to be non-coincident with other quanta. A search was made for any high energy gamma rays (greater than 675 kev) which might accompany the decay of Pt^{193}. It was concluded that any such hard quanta are emitted, if at all, with an intensity of less than 10^{-3} per disintegration. In the decay of Pt^{191} photopeaks corresponding to K X-rays and gamma ray energies of 130, 170, 220, 270, 355, 409, 539, and 620 kev were observed. Coincidences, between each gamma ray and the remainder of the spectrum were investigated. The spectrum of Pt^{191} is actually far more complex than above indicated. Distortion of the pulse height distribution and the presence of broad peaks indicated the complex nature of the spectrum.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

** Research Fellow, permanent address Gwalior (M.B.) India.

*** The Franklin Institute.

A DEVICE FOR MEASURING THE MAGNETIC FIELD IN A MASS-SPECTROGRAPH

BY

W. F. G. SWANN

The device consists primarily of two coils rotating on the same axis, one in the field to be measured and the other in a solenoid in which the field can be adjusted to produce a balance of electromotive force.

The feature of the device lies in the avoidance of any slip-ring or other contact of the kind. The ends of the coil system are connected to two cylinders which rotate with the shaft. These cylinders, whose potential varies sinusoidally, act inductively upon surrounding stationary insulated cylinders connected to a suitable amplifying system, and through it to an oscillograph. The mid-point—in resistance—of the rotating coil system is connected to the rotating axis, and this axis is connected to the "ground" of the system through a mercury connection. Any fluctuations in this connection are without significant influence upon the behavior of the system.

The whole system is enclosed in a shielding enclosure which conforms to its shape, so that no part of the insulated system, or even of the central system, is exposed to the exterior with the exception of a small turbine which is attached to the rotating shaft and is driven by an air blast.

NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

WRITING MATERIALS AND TOOLS

As every schoolboy learns—and quickly forgets—our alphabet is Roman. Perhaps the schoolboy might be induced to remember that fact a little longer if he recalled that it is the same alphabet that was used by Cicero and Caesar. Today, we are so accustomed to seeing the symbols of this alphabet reproduced in such a multitude of varieties that we are prone to forget its origin among the fierce farmers who lived along the banks of the Tiber, and the peculiar methods that had to be adopted to preserve it as a written language.

Chief among the writing tools of the earliest days was the edged pen, which was fashioned from a hollow cane or reed, and later from a quill. This pen was used to shape all the written letters during that long period when Latin inscriptions and manuscripts were done by hand, with the exception of the Roman cursive hand. This latter was written by a blunt pen called a stylus.

The reed pens displayed in the Graphic Arts Section of the Museum, it is interesting to note, have split points, like our modern steel pens.

The materials upon which these styli and pens were used to inscribe the characters were wax, papyrus, vellum, and parchment. Although it is commonly believed that papyrus was essentially an Egyptian product, it should not be forgotten that the Egyptians developed a brisk trade in it, and that many Greek and Roman manuscripts were written on papyrus. This writing material was made from the triangular stem of the papyrus plant. After the long stem had been cut into sections, the pith (which was about five inches thick) was sliced into thin strips. These strips were then laid side by side in two layers, one at right angles to the other. The whole was made to adhere closely either by an adhesive or by pressure while still wet, it is not known which. The specimen of modern papyrus shown in the Graphic Arts section was made by pressure. The sheet was then smoothed by scraping and polished with smooth shells. The sheets were from five to fifteen inches wide, but averaged nine inches. These small sheets were pasted together to make a large sheet of the required length. In the case of a long manuscript, such as a code of laws, it was formed into a long roll. Folding caused papyrus to break.

The wax tablet was subordinate to papyrus and was used for short notes or for small documents. The wax on the wooden frame could be erased and the surface used again. But wax tablets have an importance. Someone got the happy idea of fastening two or more tablets together to form what the Romans called a *codex*, and this eventually emerged as a book.

Vellum was calfskin and parchment was sheepskin or goatskin. Although we use the words interchangeably they have a distinction. Parchment had been in use before the Romans adopted it, but as the papyrus rolls on which their laws were written had become unwieldy, parchment had to be adopted as it was adaptable to the codex form that occupied less space and was much easier to handle. It had other advantages. Parchment could carry writing on both sides, which neither the wax tablets nor papyrus could do. Moreover, it was much more durable than the brittle papyrus, easier to write on, and made the writing more legible.

By the fourth century parchment was the most commonly used writing material. Because of its enduring qualities it was an ideal material for those books that had to endure through generations, but because it was not procurable in sufficient quantity, the number of books was necessarily restricted.

When parchment became the chief writing material, the quill pen became the principal writing tool. The ink, too, was improved. The Romans had introduced sepia, the black liquid extracted from the cuttlefish, in place of the soot, mixed with water and gum, that the Egyptians had used, but the older material was still most common. A slight improvement had been made by adding a small quantity of vinegar to make it set better.

Paper is supposed to have originated in China about the second century, and to have reached Europe through Arab traders about the eighth century. In spite of what would appear to be its obvious advantages, paper was not altogether welcomed. Its origin with barbaric peoples made it exceedingly unpopular among the churchmen, who were responsible for the reproduction of most books, and some rulers forbade the writing of legal documents on paper. However, the demand for more books and more accurate reproductions of manuscripts created the atmosphere for printing.

Gutenberg brought together some familiar tools and arranged them so that it became possible to print books in any numbers at much lower cost. Fortunately, the oil-based ink had been discovered by this time. The paper that was used in his time was made exclusively from linen rags, but as the output of the printing press not only increased but created a demand for further increase, linen rags proved insufficient to meet the demand for paper. The story of the vicissitudes through which paper passed before it arrived at its present stage of development, is too long to be attempted here. But the scale model papermaking machine in the Museum has produced paper from such strange sources as glass and nylon, while additives of rubber, magnesia, various plastics, and earths have added spice and variety to the experiments that have been conducted upon this useful machine.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

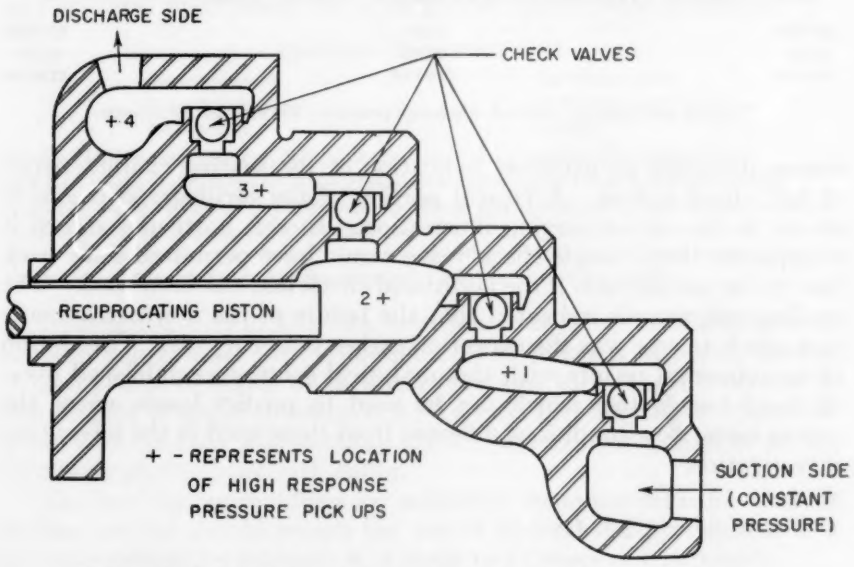
CHECK VALVE INVESTIGATION

BY

R. H. MARVIN

A research project has been conducted in the Applied Hydraulics Section of the Mechanical Engineering Division on fluid flow past the check valves of a positive-displacement pump. This investigation was carried out for the Milton Roy Company, manufacturers of controlled volume pumps. The object of the program was to determine flow and pressure relationships and study valve operation in the double check valves used on the suction and discharge sides of the sponsor's pump.

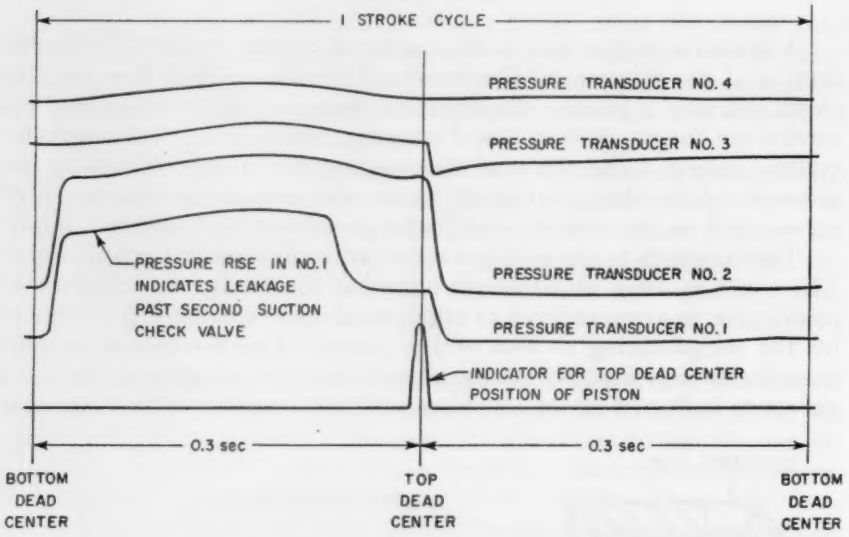
The approach to the problem was both analytical and experimental. The common form of Bernoulli equation for steady flow had to be modified in an exact analysis to take into account unsteady flow induced by the reciprocating motion of the piston. Quick-response pressure transducers were installed on the up and down stream sides of the check valves as indicated in the accompanying illustrations. The outputs of



Schematic of valve body and piston, showing location of pressure pick-ups.

these were fed to a multichannel oscillograph. Also recorded was an indication of the top dead center position of the piston. With this exact knowledge of pressure-time and displacement-time history it was possible to observe the relative timing associated with the seating of the ball checks in their seats, and evaluate the effectiveness of the two check valves in maintaining uniformly high volumetric efficiency.

The relative differences between recorded pressures gave a clear record of the pressure drops along the flow path and permitted a rational analysis of the losses in the liquid end of the pump. The pressure


1 STROKE CYCLE
PRESSURE TRANSDUCER NO. 4
PRESSURE TRANSDUCER NO. 3
PRESSURE RISE IN NO. 1 INDICATES LEAKAGE PAST SECOND SUCTION CHECK VALVE
PRESSURE TRANSDUCER NO. 2
PRESSURE TRANSDUCER NO. 1
INDICATOR FOR TOP DEAD CENTER POSITION OF PISTON
0.3 sec
0.3 sec
BOTTOM DEAD CENTER
TOP DEAD CENTER
BOTTOM DEAD CENTER

Typical oscillogram—200-psi. discharge pressure; 100 strokes per minute.

traces also gave an excellent indication of the seating characteristics of ball check valves. A typical portion of the oscillograph record is shown in the accompanying illustration. In this particular record it is apparent that a single check valve would have permitted some back flow to the suction side if the additional check had not been used. The oscillograph records indicated that the failure of the downstream suction check to seat was the exception rather than the rule. Correlation of experimental results with the analytical solutions established overall head loss factors which can be used to predict losses across the valves under flow conditions different from those used in the laboratory investigation.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, M.D., FOUNDER WILLIAM G. BATT, D.Sc., DIRECTOR

N-METHYL-2-PYRROLIDONE AS A SOLVENT FOR PYRIMIDINES

BY

HENRY A. RUTTER, JR.

Studies in this laboratory of the reactions of pyrimidines have been impeded by the low solubility of these compounds in most solvents, especially at high concentrations and low temperatures. Although anhydrous hydrogen fluoride has been found to be an elegant solvent at temperatures below 0° (JOUR. FRANKLIN INST., 259, 65 (1955)), the solvent is hazardous.

In a search for non-hazardous solvents, *N*-methyl-2-pyrrolidone* was chosen. This solvent is a heterocyclic amide somewhat analogous to the pyrimidines in structure and should therefore have solvent power for these compounds. For this reason, it was decided to investigate the effectiveness of *N*-methyl-2-pyrrolidone.

The *N*-methol-2-pyrrolidone was pipetted in increments of 5 ml. into a test tube containing 0.25 g. of the pyrimidine. After addition of each 5 ml. the tube was shaken and allowed to stand for 5 minutes. This was repeated until the pyrimidine dissolved at room temperature. In all instances 10 ml. of solvent were sufficient to form clear 2.5 per cent solutions. The compounds could be recovered by adding water to turbidity (about 50 per cent dilution), cooling, and addition of just enough water to start incipient crystallization. Adenine, guanine, and uric acid were found not to be soluble under these conditions.

Pyrimidines Found to be Soluble.

5-methyluracil	barbituric acid
6-methyluracil	2-thiobarbituric acid
2-thiouracil	alloxan

2-oxo-4-(3,4-dimethoxyphenyl)-5-carbethoxy-6-methyl-1,2,3,4-tetrahydropyrimidine
2-thio-4-(*p*-dimethylaminophenyl)-5-carbethoxy-6-methyl-1,2,3,4-thetahydropyrimidine
2-oxo-4-(3,4-diethoxyphenyl)-5-carbethoxy-6-trifluoromethyl-1,2,3,4,-tetrahydropyrimidine
2-oxo-4-(*p*-dimethylaminophenyl)-5-carbethoxy-
6-trifluoromethyl-1,2,3,4-tetrahydropyrimidine
2-thio-4-phenyl-5-carbethoxy-6-trifluoromethyl-1,2,3,4-tetrahydropyrimidine

N-methyl-2-pyrrolidone has been found to be an excellent solvent at room temperature for pyrimidines. The behavior of pyrimidines in *N*-methyl-2-pyrrolidone in chemical reactions with reagents soluble in this solvent is under investigation; for example, alkylation of pyrimidines with sodium and methyl iodide, and dehydrogenation of the tetrahydropyrimidines with sulfur.

The fact that pyrimidines are soluble at room temperature whereas purines are not should permit the use of *N*-methyl-2-pyrrolidone as a selective solvent for separation of these two classes of compounds.

* Generously supplied by the Commercial Development Department of General Aniline and Film Company, New York.

BOOK REVIEWS

MATHEMATICAL FOUNDATIONS OF QUANTUM MECHANICS, by John von Neumann, translated by Robert T. Beyer. 445 pages, 15 × 23 cm. Princeton, Princeton University Press, 1955. Price, $6.00 (paper).

Here is a book which contains precisely what its title promises. Attention is focused primarily on the rigorous, mathematical foundations which form the basis of present-day quantum mechanics.

The author utilizes the proven equivalence of wave mechanics and matrix mechanics to extract the properties which are common to these two modes of description. In matrix mechanics the eigenvalue problem is solved by obtaining a sequence of numbers, $x_1, x_2 \ldots$ which satisfies certain computation relations. In addition it is required that $\sum_\nu |X_\nu|^2$ be finite. The totality of such sequences, A, constitutes the elements of what is commonly known as a Hilbert space. In wave mechanics one obtains solutions of Schrödinger's equation. These solutions, B, are functions of continuous variables fulfilling the well known integrability condition. It is possible to establish an isomorphism between the elements of A and B. All those fundamental properties of Hilbert space which are independent of the form of the functions constitute the mathematical structure known as "abstract Hilbert space." By the use of this space, it is possible for the author to develop a rigorous unified approach. It becomes unnecessary to introduce improper functions such as the Dirac delta function which are so essential in the transformation theory approach of Dirac and Jordan.

The first chapter describes the lack of mathematical rigor in the Dirac approach and states that this can be overcome by the use of Hilbert space. The next chapter, comprising one-third of the book, discusses the properties of Hilbert space as well as the formulation of its eigenvalue problem. In chapter three all assertions of Quantum Mechanics are reduced to a statistical expectation value formula. This formula can be explained as follows: If the physical quantity $\mathfrak{R}$ is represented by an operator R and the wave function characterizing the state of the system is ϕ then the expectation of $\mathfrak{R}$ in the state ϕ equals $(R\phi, \phi)$ where (ψ, ϕ) denotes the inner product of two states ψ and ϕ considered as elements in Hilbert space. It is to be noted that the formula depends only on the state of the system and on the operator associated with the physical quantity under consideration. Chapter four then derives this formula deductively from a few general qualitative assumptions. The last two chapters deal mainly with the thermodynamics of quantum mechanical ensembles and a discussion of the measuring process.

Von Neumann's book originally appeared in 1932 as volume 38 in the series "Die Grundlehren Der Mathematischen Wissenschaften In Einzeldarstellungen" and was reprinted by Dover in 1943. The translation is a very faithful rendition of the original text, and it has the advantage of having the explanatory remarks and references appear as footnotes rather than at the end of the volume as in the original. It is regrettable, however, that the translation of this classic has been reproduced in offset type and is sold without a hard cover. There is no index! But none is really necessary. It will generally not be used as a reference book as the ideas have to be digested chapterwise. For this purpose the table of contents is quite adequate. A list of von Neumann's original contributions which form the basis of this book can be found in *Scripta Mathematica*, Volume IV, p. 90 (1936).

ELMER L. OFFENBACHER

INTRODUCTION TO THEORETICAL ORGANIC CHEMISTRY, by P. H. Hermans, edited and revised by R. E. Reeves. 507 pages, 16 × 24 cm. Houston, Elsevier Publishing Co., 1954. Price, $9.75.

This is the first English edition of a book published originally by Dr. Hermans

in Dutch in 1952. The earlier edition was based upon notes collected by the author for his own use; for the current edition the subject matter has been further systematized and greatly amplified and extended.

As pointed out in the preface: "The development of the theoretical aspects of organic chemistry has made a great advance during the last 25 years. Most textbooks of organic chemistry have taken little or no cognizance of it, while the relatively small number of existing books dealing exclusively with the theoretical aspects are either too profound or too specialized to serve as a general introduction to the subject." The author and editor have attempted to improve the situation by preparing a book which will bring together the fruitful efforts of the past two decades of work in theoretical organic chemistry. They have applied logically the principles of physical chemistry to organic reactions and organic systems and have succeeded in presenting a comprehensive view of the whole subject which will be of great value to the serious student. "The author has tried to discuss a broad field in a simple and lucid manner without striving for completeness. No more than an elementary knowledge of physics and chemistry is required by the reader and the use of mathematical formulae is reduced to a minimum. . . . Further, at the end of most chapters, a list of books is given . . . for further study. . . ."

The efforts of the author and editor have resulted in the production of a treatment of exceptional merit of this important and rapidly widening field of chemistry. The subject has been developed logically from concisely stated principles and as completely as present knowledge would seem to warrant. The information given seems to be soundly based and well up to date. The most conspicuous omission, perhaps, has been a description of the physico-chemical methods by which the results have been obtained. The inclusion of these, however, would have considerably enlarged the book; for those unfamiliar with such methods, references to work in connection with the most important developments have been provided in footnotes. References are given also to other treatments, parallel or otherwise, of many of the topics discussed. A few of the mathematical relations are briefly discussed in the appendix. Author and subject indexes are included.

The scope and sequence of the topics discussed may be judged from the chapter list, which, somewhat altered by the reviewer for the sake of brevity, is as follows: I. Classical concepts of molecular structure; II. Geometry of molecules; III. Stereochemistry; IV. Atom models; V. Bonding in molecules; VI. Polarity of molecules; VII. Molecules and spectra; VIII. Magnetic properties of molecules; IX. Mass spectrometry; X. Intermolecular attraction; XI. Ions; acids and bases; XII. Tautomerism; XIII. Intramolecular transformations; XIV. Carbonium ions; XV. Substitution in aliphatic compounds; XVI–XVIII. Addition reactions; XIX–XX. Esters, their hydrolysis and condensation; XXI. Reaction velocity and equilibria; XXII. Acid and base catalysis; XXIII. Effect of substituents on properties; XXIV. Aromatic substitution; XXV. Free radicals; XXVI. Redox (oxidation-reduction) reactions.

The book is well printed, well bound and well illustrated. It should be welcomed by teachers of organic chemistry and is well worth consideration as a textbook, especially in those courses which follow the laying of a suitable foundation of factual material in the elementary course in organic chemistry.

WILLIAM B. MELDRUM

ELEMENTS OF SERVOMECHANISM THEORY, by George J. Thaler. 282 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $7.50.

The practicing engineer or the undergraduate student who desires to obtain a working knowledge of the methods of servomechanism analysis and synthesis, but who does not intend to pursue the subject of control engineering at an advanced level, will find that this book is well suited to his needs.

The author has avoided the use of the Laplace transforms and considered only the differential equation and frequency-response function representations of servomechanism systems. However, mathematical rigor has been sacrificed to perhaps an unnecessary extent in avoiding the use of elementary

complex variable theory to derive frequency-response transfer functions.

Emphasis is placed on the conventional methods of servo compensation design employing polar and logarithmic plots. These methods are explained in a very clear step by step fashion, with abundant graphical illustrations.

Only simple linear servomechanisms are considered in detail; however, a brief informative summary of advanced topics in linear and nonlinear servomechanism theory is presented in the last two chapters. Although the Laplace transform is not employed in the analysis and synthesis, a summary of Laplace transform theories and their application is presented in an appendix. A brief discussion of servomechanism components is also presented in an appendix.

The author states in the preface that the text material is intended for a one semester senior undergraduate course. The reviewer believes that this purpose is achieved. However, without detracting from the value of the book, it might be said that for students or practicing engineers intending to pursue the servomechanisms field further, a text based on Laplace transforms and including the important aspects of complex variable theory would be more appropriate.

G. S. STUBBS

RADIOISOTOPES IN BIOLOGY AND AGRICULTURE, by C. L. Comar. 481 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $7.50.

Enthusiasm for the use of radioisotopes is still often inhibited by uncertainties of approach and lack of detailed information on their use. *Radioisotopes in Biology and Agriculture* promises to contribute profoundly to the more general application of isotopic techniques. It defines fundamental principles, describes essential methodology and delineates a wide variety of applications in the biological sciences.

C. L. Comar accumulated a broad background of experience in tracer applications while Laboratory Director and Research Coordinator of The University of Tennessee—Atomic Energy Commission Agricultural Research Program. His book profits from his own experience as well as from the cooperation of authorities in a wide variety of research fields.

Principles and procedures are described in over half of the book. An excellent discussion of biological turnover and kinetics with a minimum of mathematics is augmented by authentic typical data. These and a number of other topics are followed by a discussion of the difficulties of tracer methodology. Isotope effects, exchange reactions, radioactive impurities and radiation effects should be appreciated by all users of radioisotopes. Health physics, the tolerances and methods of protection while handling radioactive materials is a major concern of the book and thoroughly covered. These chapters are well written and should be understood by all users of radioisotopes. Special techniques for work with microorganisms, plants, laboratory and farm animals are described and illustrated.

Instrumentation and radiation analysis is effectively described. A third of the volume is devoted to specific analyses and information on currently available isotopes. In this category the book becomes a reference work, its primary attribute being the fact that it is up-to-date. Costs, catalog numbers (Isotopes Div., A.E.C.) and availabilities are important data in this section.

Chapters on autoradiography, paper and ion exchange chromatography and activation analysis appear at the end of the book. These might have followed Procedures for Radioassay since they are so intimately related.

Without doubt, Dr. Comar has made an effective contribution to isotope applications in biology. There are many laboratories, particularly in Europe, where a simple but enthusiastic description of the basic techniques in radioisotope work will be more than welcome.

A. A. BENSON

OPTICAL PROPERTIES OF THIN SOLID FILMS, by O. S. Heavens. 261 pages, illustrations, diagrams, 15 × 22 cm. New York, Academic Press, Inc., 1955. Price, $6.80.

Only in the last decade or two has vacuum technology made it possible to evaporate, reproducibly, thin films of metals on various substrata. Books and technical papers con-

cerning the physical properties of these films, such as electrical resistivity, have appeared in great number. Now, Dr. Heavens has filled a gap in the literature by devoting his entire book to the optical properties of these films.

After the introduction, the author describes various methods for the formation of thin films. Major attention is placed on the thermal evaporation process as this process gives the most reproducible results.

The next chapter discusses the structure of the films and methods for determining the structure. The method of electron diffraction is described in some detail, since by this method crystal orientation and grain size in the film may be determined. A welcome portion in the chapter is one section on the effects of the measuring device, for example, the effect of the electron beam on the film when using the electron microscope. Besides the heating effect, which can actually melt some specimens, the prolonged bombardment of the sample with oil molecules from the diffusion pump will in time poison the surface.

Chapter 4 discusses the optics of thin films. This complicated subject is treated extensively, but with a minimum of esoteric details. A section on graphical methods for treating reflection gives a physical insight to the problem.

Chapters 5 and 6 describe methods of measurement of the optical properties and the results of these measurements. The included data cover mainly only gold and silver films. An interesting comparison is made between the properties of a deposited gold film and a gold film produced by beating.

The last chapter deals with practical applications of thin films, such as the interferometer for the single layer film to the many layer type used as interference filters.

Extensive references are given at the end of each chapter covering early work as well as work done as late as 1954.

ROBERT B. CAMPBELL

PUBLICATIONS RECEIVED

APPLIED THERMODYNAMICS, by Earle B. Norris, Eric Therkelsen and Clarence E. Trent. 490 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $7.50.

PRINCIPLES OF MATHEMATICS, by C. B. Allendorfer and C. O. Oakley. 448 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $5.00.

PRACTICAL PHYSICS, by Marsh W. White, Kenneth V. Manning and Robert L. Weber. Second edition, 484 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $5.50.

NEW METHODS IN ANALYTICAL CHEMISTRY, by Ronald Belcher and Cecil L. Wilson. 287 pages, 15 × 22 cm. New York, Reinhold Publishing Corp., 1955. Price, $5.50.

LA THEORIE DES FONCTIONS DE BESSEL, by Gerard Petiau. 477 pages, 17 × 25 cm. Paris, Centre National de la Recherche Scientifique, 1955. Price, 2500 fr.

THERMODYNAMICS, by Joseph Louis Finck. 224 pages, diagrams, 16 × 24 cm. New York, Bookman Associates, 1955. Price, $7.50.

EUROPEAN FIREARMS, by J. F. Hayward. 53 pages, 34 plates, 19 × 25 cm. New York, Philosophical Library, 1955. Price, $7.50.

HIGHWAY TO THE NORTH, by Frank Illingworth. 293 pages, illustrations, 15 × 22 cm. New York, Philosophical Library, 1955. Price, $7.50.

CURRENT TOPICS

Silicon Power Rectifier.—A tiny new electronic device made of extremely pure silicon shows promise of reducing costs on a large scale in many industries using electric power, Bell Telephone Laboratories announced recently. The simple device—a silicon power rectifier—converts alternating current into direct current, an essential step in the operation of telephone systems.

Industries and homes also depend upon rectifiers to convert standard alternating current to direct current for such diverse purposes as running machinery and operating television sets. It is not economically feasible to generate and transmit direct current.

Despite a trend in recent years toward smaller and more efficient components, power rectifiers have remained large, limited in efficiency and in need of bulky cooling equipment to prevent overheating.

Bell scientists believe that the new power rectifier will far surpass any yet made of silicon or other material. They expect it may have an almost unlimited life span, and will be capable of operating continuously at temperatures up to 400° F.

These advantages are expected to open up many new uses in telecommunication, heavy industry and in military applications. The rectifier's efficiency—more than 98 per cent of the theoretical limit—was made possible by recent work of M. B. Prince, a Bell Laboratories physicist.

Two of the new rectifiers, when made about the size of peas, linked together, and mounted on a cooling fin, will furnish more than 20 amp. d.c. at 100 volts. This amounts to 2000 watts—with only 20 watts lost through heat.

Only one ampere or less is needed to run a radio or television set, or to operate a group of telephone switching relays. To charge storage batteries used in automobiles and telephone central offices or to operate electric motors, from 10 to more than 100 amp. may be needed.

Highly purified silicon is a costly material when bought in pound quantities, but such minute amounts are used in the new rectifier that the material becomes relatively inexpensive to use. A rectifier the size of a pea, for example, contains a silicon wafer only five one-thousandths of an inch thick and one-tenth of an inch square.

Even smaller rectifiers have been made and will be desirable for use in the telephone system and in electronic computers.

Larger rectifiers are necessary for other uses where greater amounts of power must be handled. The size of the wafer would vary, just as the size of a copper wire ranges from thin to heavy depending upon the current that must be carried.

The new silicon rectifiers provide 5000 times more current than conventional rectifiers of the same size, thus permitting miniature operating units. Even so, the newest techniques may be applied to large sizes and thereby provide entirely new current and power possibilities for industry. On a machine the size of a locomotive, for example, significant amounts of space may be saved by the new rectifiers.

The silicon power rectifier grew out of the same broad research program

at Bell Laboratories which produced the transistor—a tiny, long-lived amplifying device—and the Bell Solar Battery—first device to convert sunlight into substantial amounts of electricity.

The techniques created at the Laboratories for producing this silicon device involve the controlled introduction of impurity elements into the layers near the surface of a thin slice of silicon.

Treatment with vapors at high temperatures causes a minute trace of the impurities to enter the crystalline structure of the silicon. Before this process begins the silicon must first be so pure that there is less than one impurity atom per billion silicon atoms.

The controlled introduction of impurity atoms forms in the silicon a p-n (positive-negative) junction, which is the heart of the transistor. C. S. Fuller and G. L. Pearson, of Bell Telephone Laboratories, announced the application of the techniques to power rectifiers more than a year ago.

Development of these techniques as applied to a wide variety of silicon devices is being intensively pursued by K. D. Smith, C. J. Frosch and their associates.

Production plans for the new silicon power rectifier are being worked out with the Western Electric Company, manufacturing unit of the Bell System. Production will start soon for both Bell System and military uses.

Drill that Can Split Hairs.—General Electric's instrument makers have gone one-up on the old bromide about splitting hairs. They have drilled holes in hair and then threaded wire through the holes. The company's General Engineering Laboratory uses a one-mil (0.001-in.) drill, too small to be seen by the naked eye and so delicate it could be snapped or bent by accidental contact with a piece of paper tissue.

It takes a microscope and a steady hand to thread such a tiny hole—but GE instrument makers have succeeded in lacing one-mil wire, a thousandth of an inch in diameter, through something that is even smaller than human hair—a strand of nylon stocking.

The laboratory uses this microscopic drill for such jobs as drilling tiny fuel injection nozzles, making orifices in leak discs that control the flow of gas into a vacuum chamber, and making apertures for electron beams in sensitive X-ray equipment.

Contrary to standard shop practice, drill operators must slow down the drill press to approximately 1800 rpm. when using the one-mil size. In drilling operations, the usual formula is the larger the drill, the slower the speed, but the formula works in reverse on the tiny drills.

The ultra-sensitive drill press is located in the laboratory machine shop, where highly skilled instrument makers are "on call" for many unusual tasks in connection with experimental developments.

Highly Selective Electronic Filters.—A new type of electronic filter, Model EPN-10-A, providing a higher degree of selectivity than has ever been possible before, has been announced by L. M. Electronics, Inc., Los Angeles, Calif. These filters embody a new kind of electronic filter circuit making possible degrees of selectivity heretofore obtainable only by means of mechanical filters. They can be obtained with band-pass or band-elimination characteristics, or with both characteristics combined in one unit, so that either can be selected by means of a switch on the front panel.

Q values of the order of 1000 can be obtained. This is far in excess of the modest values of Q to be found in ordinary selective laboratory amplifiers. The filters described here were originally developed to take care of the severe equalization problem met with in vibration test equipment. Because of the low damping often associated with mechanical oscillating systems, very sharp resonances occur. The resonance peaks of the equalizer must be equally sharp, and the sharpness, or selectivity, must be continuously adjustable, which is also true of the resonance frequency.

These new filters will have many uses in other fields, as well as in the electronic laboratory in general. The resonance frequency is continuously variable from 30 to 3000 cps., and the selectivity can be continuously adjusted from a Q of zero up to values of 1000 or more. The peak height and notch depth can be adjusted independently of the Q value. High frequency and amplitude stability is assured by the use of highest-grade component parts, and by utilizing the latest techniques in electronic design. The circuit (patent applied for) making possible these high values of Q is surprisingly simple, and its stability is of a high order. The noise level is low.

As the filter employs electron tubes, the signal amplitude must be kept below a certain level in order to avoid overloading, which would upset the normal operation of the filter. Any signal up to 1.5 volts peak amplitude will give proper operation.

Pocket-Size Measuring and Calculating Device.—A new precision measuring and calculating instrument for skilled craftsmen, engineer and scientists is being manufactured by Jet Air Engineering Corporation, San Diego, Calif.

Known as the Caliputer, the pocket-size stainless steel device combines the functions of a vernier caliper, a depth gage, and a slide rule computer in a single inexpensive unit. Each Caliputer is packaged in a vinyl plastic scabbard, on which a company trademark and advertising can be printed. The Caliputer itself, made of satin finish type 302 stainless steel, is precision-etched and measures $4\frac{1}{2}$ in. long, $1\frac{1}{2}$ in. wide, and $\frac{1}{16}$ in. thick.

In addition to combining several commonly used devices in a single unit, the Caliputer provides direct readings of circumferences and cross-section areas, as well as permitting easy calculation of other required information. From the Caliputer, for example, a machinist can read directly the correct speed setting for a lathe in revolutions per minute by setting the rule to the diameter of the stock being used and by supplying the proper feed rate.

Readings of the $4\frac{1}{2}$-in. caliper and depth gage are accurate to one-thousandth of an inch, and the linear scale is graduated to thirty-seconds of an inch, and the slide rule performs all multiplication, division, square and square root calculations.

Super-Power X-Ray Machine.—The world's most powerful and versatile "crystallographic" X-ray machine —which is one especially designed to study the atomic structure of solid matter—has been put into operation at the Westinghouse Research Laboratories.

Dr. J. A. Hutcheson, vice president and director of Westinghouse research said the new apparatus is up to six times more powerful than comparable machines, permits a sixfold reduction in the exposure time for X-ray photographs, and yields types of radiations

entirely beyond the capabilities of ordinary X-ray equipment.

"This X-ray machine is strictly a tool for fundamental research," Dr. Hutcheson declared. "Its power, speed, and flexibility make it an ideal instrument for investigating such basic problems of matter as the influence of atomic arrangement on magnetism, the mechanisms of metallurgical reactions, and the effects of temperature on the atomic architecture of crystals."

The designer of the machine, Dr. Abraham Taylor, a research scientist in the magnetics and solid state physics department, supervised its construction at the Westinghouse Research Laboratories.

Heart of the new apparatus, Dr. Taylor explained, is a radically improved type of crystallographic X-ray tube which yields a narrow, intense beam of X-rays. Directing the beam through a crystalline material and allowing the rays to then fall on a photographic film gives a pattern that tells the arrangement of atoms in the crystal.

The usual X-ray tube for this purpose yields X-rays activated by a continuous current of 15 or 20 milliamperes at 45,000 volts. The X-rays in the new tube result from a continuous current of 120 milliamperes at 45,000 volts—at least six times as much current and power. Such additional power is especially important in research because it permits X-ray photographs in one-sixth the usual time, collapsing months of normal X-ray work into a few days or weeks.

The high power output of Dr. Taylor's X-ray tube comes from a unique, water-cooled rotating metal anode, or positively charged electrode. This anode, about five inches in diameter, resembles a hollow metal wheel and rotates about 1000 rpm. X-rays are generated when a stream of electrons from a special type of cathode, or negatively charged electrode, smash head on into the rim of the whirling anode. Cooling water is pumped through a cavity in the anode. The high power output of the tube is possible because the whirling anode always presents a cool, fresh surface to the electron stream. At such power a stationary anode, even if water cooled, would soon vaporize under the terrific heat generated by impact of the electrons on their target.

Dr. Taylor explained that the new Westinghouse X-ray machine is designed to give different types of X-rays almost on a moment's notice, and described this as an important advantage of the machine for X-ray research. When bombarded with electrons, different metals give rise to different kinds of X-rays. The new X-ray tube is provided with extra anodes, each plated with a different metal. These anodes can be interchanged within a few mintes, thereby providing a variety of X-rays which can be selected by the research scientist with speed and ease. The usual crystallographic X-ray tube has a stationary nonreplaceable anode, and separate tubes (each costing several hundred dollars) must be used to obtain different kinds of X-rays.

"We have been able to produce new and unique radiations by using anodes plated with metals such as gallium," Dr. Taylor said. "This metal has such a low melting point that is is impossible to use it as a target in any sort of conventional X-ray tube."

The Westinghouse scientist stated that the new X-ray machine has made possible the study of atomic arrangements at a new range of high and low temperatures. Because of the great intensity of the X-ray beam, photographs which ordinarily require an

exposure of one or two hours can be made in a matter of minutes. Thus, X-ray "snapshots" of metals as hot as 3000° F. have been taken by Dr. Taylor, making it possible to study a metal before it vaporizes or undergoes some chemical change at the elevated temperature.

The new X-ray tube is constructed of stainless steel with walls ¼ in. thick. Like the rotating anode, these metal walls are cooled with water, since electrons rebounding from the target would otherwise heat the walls red hot within a few seconds. Inserted in the walls of the tube are four "windows" made from beryllium metal foil five-thousandths of an inch thick. Through these "windows" the super-power X-rays are beamed.

The Westinghouse Research Laboratories are already making intensive use of the new X-ray equipment, Dr. Taylor reported. A special type of X-ray image amplifier tube called a FLUOREX—itself a development of the Research Laboratories—is being provided for the machine. This will permit direct observation of the patterns formed by the atoms in crystals without the necessity of taking the usual X-ray photographs.

Dr. Taylor said that the machine is also equipped with an adapter which gives an X-ray beam only 400 millionths of an inch in diameter—about one-tenth the diameter of a human hair. This thin, intense beam approaches the dimensions of the tiny crystallites which make up a crystal of metal, and permits the investigation of individual crystals with an exactness not possible with a beam of larger size.

Improved Adhesive Bandages.—The plastic age has brought many improvements to products and when plastic film was used as backing for adhesive bandages, it made the bandages stay firmly on even though repeatedly immersed in water. However, it introduced the annoying attribute of maceration, or whitening of the skin.

This has been solved by a new plastic bandage with a fast-drying pad thereby virtually eliminating skin crinkling and whitening, introduced by Johnson & Johnson (New Brunswick, N. J.). The probably patentable feature providing this remarkable result is now incorporated in BAND-AID Plastic Strips which soon will be available everywhere.

The frequent placing of hands in water by wearers of all types of plastic adhesive bandages which thoroughly soaks the pad on the bandage has caused a condition of maceration, particularly after a day or more of wearing the bandage. Heretofore the waterproof plastic backing retarded the drying of the pad—not permitting it to aerate—and the water remained trapped in the gauze. Moreover, the presence of water for a period of time over a small scratch or cut retards the process of healing.

The old-type cloth adhesive bandage had not conformed closely to the contour of the skin; thus it allowed for evaporation. The plastic bandage, on the other hand, gave better protection and had conformability. However, the fact that it stayed on longer, even though the hands were repeatedly immersed in water, accentuated the skin whitening and crinkling.

As research proceeded, perforations in numerous patterns and of various sizes were tried experimentally. Clinical tests were carried on with groups totalling more than a thousand persons wearing at a time eight different versions of the experimental adhesive bandages over a 24-hr. period. Accurate records were kept and literally

hundreds of color photographs were made.

After thousands of tests and detailed studies, the solution appeared. It was found that tiny ventilating perforations distributed for a uniform evaporating surface permitted a vast increase in the moisture vapor removal rate without reducing waterproofness.

Electronic Brain Being Tested at Belvoir.—An electronic computer which breezes through simultaneous equations and least squares has been developed by the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Va., and the Monrobot Laboratory of Morris Plains, N. J.

Called Monrobot V, it will speed-up the preparation of military maps by reducing complicated engineering calculations to a minimum. Tests have proved that it makes figuring faster, easier, cheaper and more accurate than ever before.

The computer resembles a large office desk and weighs about 1500 lb. Equipped with shock absorbers to cushion vibration, it is rugged enough to be used in the field. For easy loading into a mobile laboratory, it can be separated at the middle into two sections.

Easy maintenance under field conditions is facilitated by the use of pluggable units, and a low power consumption of about 5 kilowatts makes it ideal for the Army's portable units. The machine is designed to withstand dust and extreme changes in temperature and humidity, and has its own built-in cooling system.

Simple to operate, it also has a built-in keyboard which permits the entering of problems in their algebraic form. It is equipped with punched-tape facilities for automatic operation. Results may be printed on a paper tape eight-inches wide, or punched out in the form of perforated tape. Its electronic gray matter can turn out trigonometric calculations to ten-place accuracy.

Monrobot V will undergo engineering tests at Fort Belvoir, where it will compute traverse, trig-traverse and three point problems; triangle, position and azimuth computations; geographic to UTM grid coordinates; spherical excess; trigonometric elevations; geodetic adjustment of trilateration, least squares, and other related surveying problems.

"Building Block" Concept Switchboard Design.—The General Electric Company has announced a new basic "circuit" concept in switchboard design. The new approach involves use of pre-engineered and time-tested basic circuits in the design of control switchboards.

H. Frank Hentschel, general manager of G.E.'s Medium Voltage Switchgear Department, Philadelphia, which developed the new concept, said there are eleven basic circuits which cover applications for the great majority of switchboard users. Each circuit includes basic instruments, meters, control devices and relays necessary to perform a desired function.

Components of a circuit group on the control and relay sides of switchboards are arranged directly opposite each other for operating simplicity. As a result, switchboard operators can easily visualize circuits.

Other advantages noted by Mr. Hentschel include, saving of long hours of detail planning, reduction of over-all procurement time and effort, and determination of switchboard size before orders are placed.

The eleven functional basic circuits of the new design concept are: (1) a-c. feeder with overcurrent protection; (2) a-c. feeder with directional overcurrent protection; (3) power transformer; (4) tie line with wire pilot differential protection; (5) transmission line with carrier current directional comparison protection; (6) transmission line with directional distance protection; (7) small a-c. generator; (8) large a-c. generator; (9) unit generator-transformer; (10) bus differential protection; and (11) synchronizing equipment.

A basic circuit switchboard can be selected easily from specification sheets which are available for each of these circuits. These sheets list metering, instruments and control elements of the circuits, show a one-line diagram of the circuit, and give the size and physical layout of the board.

Provision is made for the addition of optional elements to the sheet, to satisfy commonly used variations.

The new concept provides custom switchboards using basic circuits as building blocks, according to Mr. Hentschel. The same principles of standardization which have resulted in sweeping economic and operating advantages in switchgear should meet with the same wide acceptance when applied to switchboards, he predicted.

New Silver Home Projection Screen Material.—A new "silver" screen material for home movie projection is now being produced by an improved manufacturing technique developed by the Du Pont Company's Fabrics Division.

Called "Fabrilite" silver projection screen material, it is an aluminized vinyl coated fabric with greater durability than beaded projection screening. It also provides better reproduction due to superior light reflection properties, sharper definition and more uniform projection surface. Moreover, the washable screen will not abrade with use.

Offshore Radar Platforms.—The first of several offshore radar platforms, nicknamed "Texas Towers" because of their resemblance to offshore oil drilling rigs in the Gulf of Mexico, was recently launched by Bethlehem Steel at its Quincy, Mass., shipyard.

Security regulations prevent any lengthy description of the purposes and possibilities of the radar islands. However, it can be stated that the stations will be located along the Atlantic Coast from New York to Newfoundland, 80 to 100 miles at sea, and will apparently be used to give warning of unfriendly craft approaching our shores.

Each island will be of sufficient size to support electronic equipment, provide a helicopter landing, and housing for a crew. The first platform is supposed to be in position not later than August 15. The deadline is necessitated by the fact that the installation cannot be made during the hurricane season.

To support the platform, which is triangular in shape, it was found that waves of the size selected for design criteria, exert terrific force on any supporting element in the water. This led to the choice of three supporting legs—the minimum number that will support a platform without cantilevering. The three legs, approximately 150 ft. in length and 10 ft. in diameter, will take the form of heavy columns filled with reinforced concrete.

Protection of these structures against corrosion is a major consideration. It has been determined that painting, combined with cathodic protection, is

ample for all surfaces except those at the tidal and spray levels. In this critical area, the specifications call for a 35-ft. length of Lukens 10 per cent Monel-clad steel, the same thickness as the balance of the columns. Monel was selected because of its excellent resistance to the corrosive effect of salt water, salt air and salt spray. Also, along the outer circumference of the sheathing, designers specified pure Monel buffers to protect against floating objects which might surge against the piling.

Although the first tower will not have cross-bracing between the legs, such braces will be necessary on future radar platforms located in deeper water. For this application, the designers have specified Monel sheathing for about 75 ft. on each of six cross-braces. In addition to the sheathing on the braces, the pins which tie the links together under water will be made of corrosion resistant Monel with the same material fastening the pins in position.

"Wolmanized" Lumber Defeats Termites.—Home owners will be saved millions of dollars a year through the use of a fifty-year old industrial product, offered by Frost, the Forest Products Division of Olin Mathieson Chemical Corp. The product, "Wolmanized" treated lumber, is available to home owners for use in sills, posts and girders, exterior siding, floor joists, bridging and headers, subflooring and skirting below the first floor level.

"Wolmanized" treated wood is lumber that has been pressure impregnated with "Wolman" Preservative Salts chemicals to make it termite-proof, rot-proof, leach-resistant, fungi-proof, and fire resistant. The chemicals are "sudden death" to termites.

"Wolmanized" treated lumber is manufactured this way:

A charge—or load—of lumber is placed in a pressure cylinder made of steel. A vacuum of 28 in. of Mercury (30 in. is absolute vacuum) is "pulled" to remove air from wood cells and make the lumber receptive to treatment. The chemical solution, "Wolman" wood preservative salts, is heated to 140 degrees fahrenheit and admitted to the cylinder. When the cylinder is full of solution the vacuum is broken and a pressure of 175 psi. is applied and held while the solution is forced deeply into the lumber. After this the pressure is broken, solution withdrawn and the lumber is dried in automatic kilns, becoming "Wolmanized" treated wood. All "Wolmanized" treated lumber is pressure-treated.

"Wolmanized" treated lumber has these features: (1) it is completely odorless; (2) it can be handled without gloves since it is non-irritating; (3) it can be painted as untreated lumber; and (4) it can be easily glued.

About fifty years old, "Wolmanized" treated lumber was first used in 1905 in the Silesia mine fields in Germany.

Technological advances in manufacturing have resulted in considerable cost savings, so that the average consumer can now be offered "Wolmanized" treated lumber at a low price. Prior to these manufacturing advances, "Wolmanized" treated lumber was considered too high priced for average home use, but low priced enough to be a good investment for industry and government.

New Insulating Enamel.—A major step toward smaller, longer-lived, and more efficient electrical equipment has been made by scientists at the Westinghouse Research Laboratories

through the development of a new insulating enamel for copper wire.

Laboratory tests on electric motors insulated with the new enamel show that the motors can operate continuously for 10 years at a temperature of 325° F. without damage to the insulation. This length of time is equivalent to the normal operation of a refrigerator motor for 30 years, or a washing machine motor for about a century.

Announcement of the new wire enamel was made by Dr. Jack Swiss, manager of the insulation department at the Westinghouse Research Laboratories. He named F. A. Sattler and S. H. Langer, of the insulation department, as developers of the new enamel.

"This new enamel, which we call simply 1267, is a modified polyester-type resin containing about 20 percent silicone and represents the culmination of 10 years of research and development of polyester-type wire coatings," Dr. Swiss said. "By incorporating silicones into these materials, we have now produced an insulation which can withstand higher temperatures for longer periods of time than any wire enamel which does not contain silicones."

The Westinghouse scientist declared that raising the limiting temperature of an electric motor to 325° F. will mean a reduction in the size of present motors.

"Using this new enamel to replace the glass insulation normally required for high temperature motor windings would reduce the thickness of insulation by 50 percent," Dr. Swiss said. "This means we could make a standard motor of, say, 7½ horsepower no larger in size than the average 5-horsepower motor of today."

Dr. Swiss revealed that the new insulating enamel has been used for more than a year in pilot scale manufacture of Westinghouse motors. He said that it has also seen service in certain transformers and in the coils of high-temperature relays. The enamel has been successfully applied to all sizes of copper wire from gauge 40—about three-thousandths of an inch in diameter—to gauge 10—over one-tenth of an inch in diameter.

"In all respects," Dr. Swiss said, "it has given excellent results. This can be traced to the fact that we have added the superior heat resistance of the silicones to the outstanding chemical and mechanical properties already possessed by polyester-type wire enamels."

Among these properties Dr. Swiss cited the fact that polyester-type enamel is at least three times better than conventional wire coatings in its resistance to being dissolved by Freon 12 and 22, the refrigerants found in household refrigerators and home air conditioners. Such resistance is highly desirable because these appliances use electric motors which are hermetically sealed inside the refrigerating mechanism and are exposed to the solvent action of the refrigerant at all times.

The new enamel also has unusual resistance to flow under pressure, which occurs when enameled wire is tightly wound and wedged into the slots of an electric motor. This pressure causes the wire to eventually cut through less resistant insulation and "short out."

THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA

welcomes as members all those interested in its purposes and its activities

ANNUAL MEMBERS

Sustaining	$50.00
Active Family	20.00
Active	15.00
Active Non-Resident (50 miles or more from Philadelphia)	7.50
Associate Family	10.00
Associate	5.00
Student (14 to 25), with Library privileges	3.00
Student (under 25), without Library privileges	2.00

LIFE MEMBERS

Active	$300.00
Active Non-Resident (50 miles or more from Philadelphia)	100.00
Associate	100.00

PRIVILEGES

Free admission to the Museum, Planetarium, and Institute Lectures is granted to all members and to the families of Sustaining, Active Family, and Associate Family members.

The Institute News, which includes news items about the Institute as well as announcements of meetings and lectures, is sent to all members.

The *Journal of The Franklin Institute* is sent to Sustaining, Active Life, Active Family, and Active members.

Use of the Library is granted to Sustaining, Active Family, Active, and Active Non-Resident, as well as to the $3.00 Student members.

THE FRANKLIN INSTITUTE **Benjamin Franklin Parkway, Philadelphia 3, Pa.**

Gentlemen: I desire to contribute to the work of The Franklin Institute by enrolling as ..... Member, for which I enclose payment of $..... the amount due per annum.

NAME.....
(Please Print)

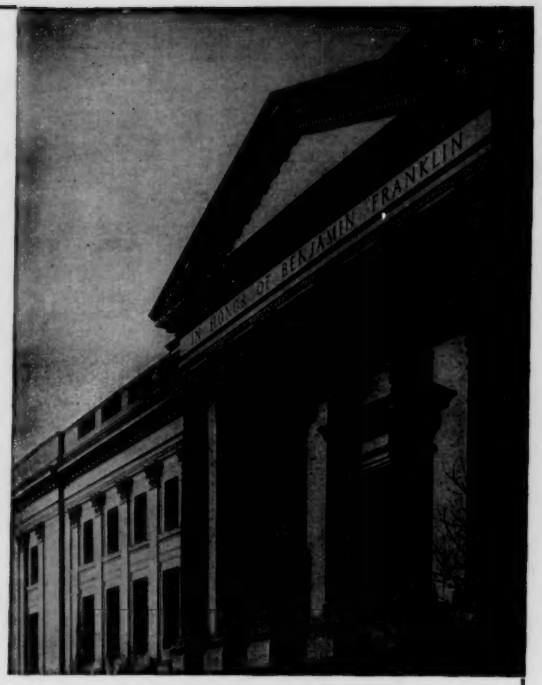
ADDRESS.....

.....

.....

SIGNATURE.....

The Franklin Institute Laboratories for Research and Development



One of the nation's largest industrial research laboratories

Where a fully qualified staff of engineers and scientists works as a competent and versatile research team—bringing a fresh scientific approach to problems of materials, methods and machines of modern industry.

in the fields of

Chemistry and Physics • **Electrical Engineering**
Mechanical Engineering • **Solid State Physics**

THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA

BENJAMIN FRANKLIN PARKWAY AT 20TH **PHILADELPHIA 3, PA.**